HUT , Institute of mathematics — Gripenberg
Mat-1.196 Mathematics of neural networks
Exercise 10
19.3–5.4.2002

1. Show that if C is a $d \times d$ symmetric matrix with nonnnegative eigenvalues, and V_0 is a $m \times d$ matrix, then the matrix $I - V_0 V_0^{\mathrm{T}} + V_0 \mathrm{e}^{Ct} V_0^{\mathrm{T}}$ is invertible for all $t \geq 0$.

2. Suppose that C is a $d \times d$ symmetric matrix with nonnegative eigenvalues, and that V_0 is a $m \times d$ matrix. Show that

$$P(t) = \mathrm{e}^{Ct} V_0^{\mathrm{T}} (I - V_0 V_0^{\mathrm{T}} + V_0 \mathrm{e}^{2Ct} V_0^{\mathrm{T}})^{-1} V_0 \mathrm{e}^{Ct}, \quad t \geq 0,$$

is the solution to the equation

$$P'(t) = P(t)C + CP(t) - 2P(t)CP(t), \quad P(0) = V_0^{\mathrm{T}} V_0, \quad t \geq 0.$$

3. Construct an neural network and an algorithm for updating the weights and thresholds such that if the inputs $\mathbf{x}_n$ are (independent) random vectors with expectation value $E(\mathbf{x})$, then the output $\mathbf{y}_n \approx \mathbf{x}_n - E(\mathbf{x})$ when $n \to \infty$.