Aalto University

MS-A0401 Foundations of discrete mathematics

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Exercise 1

7.9–11.9.2015, week 37

Return your solutions to the P-questions and answer the S-questions not later than 14.9.2015 at 16.

Remember to write your name, student number and group!

P1. Express the following sets in the form $\{$ *expression* : *condition* $\}$:

(a) $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$.

(b) $\{\ldots, -8, -3, 2, 7, 12, 17, \ldots\}$.

(c) $\{3, 6, 11, 18, 27, 38, \ldots\}$.

P2. Express the following statements using AND, OR, NOT, $\rightarrow$, $\forall$, $\exists$, $\in$, $\mathbb{R}$ and $\mathbb{Z}$ (where $\forall$ is the universal quantifier, $\exists$ is the existence quantifier, $\mathbb{R}$ is the set of real numbers and $\mathbb{Z}$ is the set of integers) as well as normal mathematical notations and parentheses:

(a) "If x is a real number but not an integer then $x \cdot 3$ is not an integer either."

(b) "For every integer y there is an integer x so that $y = 2 + x$."

(c) "There is a negative real number x so that for all integers y it holds that $y < 2 \cdot x$ or $y > x$."

Which of these statements are true?

P3. Show, using induction that

$$\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \geq 1.$$

Hint: When one has to show that two expressions give the same result it is often (both doing caluculations with a computer or with pen and paper) easiest to show that their difference is 0.

P4. If X is a set, then $\mathcal{P}(X)$ is the set of all subsest of X, that is $A \in \mathcal{P}(X)$ if and only if $A \subseteq X$. If now X and Y are two sets, is it always true that $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$? Give reasons for your answer!

P5. Prove the following claims:

(a) If a and b are integers and $(a^2 - 4 \cdot b) \cdot b^2$ is odd, then a and b are both odd.

(b) If a, b, and c are integers, $a^3 \mid b$, and $b^2 \mid c$, then $a^6 \mid c$, where $m \mid n$ means that the integer n is divisible by the integer m, i.e., there is an integer k such that $n = k \cdot m$.

Hint: Use a direct proof for one of the claims and a proof by contradiction (or contrapositive proof) for the other.