HELSINKI UNIVERSITY OF TECHNOLOGY
Institute of Mathematics
Mat-1.500 Martingale Theory, Spring 2003

Exercise VII
13.02.2003
Londen/Leskelä

1. Exercise E12.2 on page 236.

2. Exercise E12.1 on page 235.

3. Let $\{Y_n\}_{n=1}^{\infty}$ be a sequence of nonnegative independent random variables such that $\mathbb{E}(Y_n) = 1$. Let $\mathcal{F}_0 = \{\emptyset, \Omega\}$, $\mathcal{F}_n = \sigma(Y_1, \ldots, Y_n)$. Define $X_0 = 1$, $X_n = \Pi_{k=1}^{n} Y_k$.

   a) Show that $X_n^{1/2}$ is a supermartingale.

   b) Assume $\Pi_{k=1}^{\infty} \mathbb{E}(\sqrt{Y_k}) = 0$. Study the convergence and limit of $\{\sqrt{X_n}\}_{n=0}^{\infty}$, and of $\{X_n\}_{n=0}^{\infty}$. Is it true that $X_n = \mathbb{E}(X|\mathcal{F}_n)$ for some $X \in L^1$, $\forall n$?

   c) Assume $\Pi_{k=1}^{\infty} \mathbb{E}(\sqrt{Y_k}) > 0$. Show that $\{\sqrt{X_n}\}_{n=0}^{\infty}$ is a Cauchy-sequence in L^2.

4. Let $\{X_n\}_{n=0}^{\infty}$ be a supermartingale such that $\mathbb{E}(X_n)$ is constant. Show that X_n is a martingale.

5. Exercise E10.2 on page 232.