

1. Exercise E10.3 on page 232.

2. Exercise E10.4 on page 232.

3. Exercise E10.5 on page 233.

4. Exercise E10.7 on page 233–234.

5. Let $\{M_n\}_{n=0}^\infty$ be a martingale on $(\Omega, \mathcal{F}, \{\mathcal{F}_n\}_{n=0}^\infty, \mathbb{P})$ such that $|M_n(\omega)| \leq K$ for all n , almost surely. Define

$$X_n = \sum_{k=1}^n \frac{1}{k} (M_k - M_{k-1}).$$

Show that X_n is an $\{\mathcal{F}_n\}$ -martingale which converges almost surely and in L^2 .