HELSINKI UNIVERSITY OF TECHNOLOGY
Institute of Mathematics
Mat-1.500 Martingale Theory, Spring 2003

Exercise V
30.01.2003
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1. Exercise E9.1 on page 231.

2. Exercise E9.2 on page 231.

3. Prove Theorem 9.7.(g) (cDOM) on page 88, assuming (cFATOU) known.

4. Let the random variable $X \in L^2(\Omega, \mathcal{F}, P)$ and let $\mathcal{G} \subset \mathcal{F}$ ($\mathcal{G}$ a sub-σ-algebra of $\mathcal{F}$). Define
$$\mathrm{Var}(X|\ \mathcal{G}) = \mathbb{E}[\ (X - \mathbb{E}(X|\ \mathcal{G}))^2|\ \mathcal{G}].$$
Show that
$$\mathrm{Var}(X) = \mathbb{E}(\mathrm{Var}(X|\ \mathcal{G})) + \mathrm{Var}(\mathbb{E}(X|\ \mathcal{G})).$$

5. Let X, Y be two independent random variables taking values in $\{0, 1\}$ such that $\mathbb{P}(X = 1) = p$ and $\mathbb{P}(Y = 1) = p$, $p \in (0, 1)$. Define
$$Z = 1_{\{X+Y=0\}}.$$
Compute $\mathbb{E}(X|Z)$ and $\mathbb{E}(Y|Z)$. Are these independent?