On John's problem on freely floating bodies

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Abstract

The linearized equations governing the small amplitude motion of the coupled system consisting of rigid objects floating in an incompressible inviscid liquid were first derived in John [1]. Since then the problem has been largely investigated when the obstacle is fixed, but for freely floating bodies it was practically abandoned after the pioneering work of John [2].

The linearized water-wave problem has a great number of variants, many of them related with classical problems of hydrodynamics such as ship waves or edge waves or the scattering and radiation problems. Here, we are concerned with the problem of water wave trapping associated with free oscillations of a heavy liquid around a freely floating (or fixed) object. Note that if a structure supports trapped modes then the solution to the corresponding water-wave problem with suitable radiation conditions at infinity is non unique.

Unlike the water-wave trapping by fixed obstacles which corresponds to solving a linear eigenvalue problem in the frequency domain, a freely floating object turns the trapping problem into a quadratic operator pencil due to the additional equation governing the motion of the body itself. At the same time, one is faced with a coupled system composed of a scalar equation for the velocity potential and an algebraic system for the complex amplitude vector of the rigid body displacements.

Recently, we constructed a scheme that reduces the coupled quadratic eigenvalue problem to a linear self-adjoint operator in Hilbert space and, consequently, allows one to derive a sufficient condition guaranteeing the existence of trapped modes, [4]. In this talk, we will outline this scheme after first giving a physical motivation to the problem. Besides, we shall show in detail how similar geometric sufficient condition can be derived for a fixed obstacle.

References

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[4] Nazarov, S.A. & Videman J.H. 2011 Trapping of water waves by freely floating structures in a channel, *Proc. R. Soc. London A*, **467**, 3613–3632.