# Boundary element methods for time-fractional diffusion equations

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## Abstract

In this paper we discuss the numerical solution of the space-time boundary integral equation

$$S_\Gamma u_\Gamma(x,t) = \int_0^t \int_\Gamma u_\Gamma(y,\tau) E(x-y, t-\tau) \mathrm{d}s_y \mathrm{d}\tau = f(x,t), \ x \in \Gamma, 0 < t < T,$$

where $\Gamma$ is a smooth plane curve. The kernel of the integral operator,

$$E(x,t) = \frac{1}{\pi} t^{\alpha-1} |x|^{-2} H_{12}^{20}(\frac{1}{4}|x|^2 t^{-\alpha}|_{(1,1),(1,1)}^{(\alpha,\alpha)}), \ 0 < \alpha \leq 1,$$

is the fundamental solution of the time-fractional diffusion equation

$$\begin{aligned} \partial_t^\alpha \Phi - \Delta \Phi &= 0, \ \text{in } Q_T = \Omega \times (0,T), \\ B(\Phi) &= g, \ \text{on } \Sigma_T = \Gamma \times (0,T) \\ \Phi(x,0) &= 0, \ x \in \Omega, \end{aligned} \tag{1}$$

where the boundary operator $B(\Phi) = \Phi|_{\Sigma_T}$, and $\partial_t^\alpha$ is the Caputo time derivative of the fractional order $0 < \alpha \leq 1$.

We shall consider the spline collocation method for the numerical approximation of the solution on quasi-uniform meshes with tensor product splines as the approximation space. We will show that the spline collocation method is stable in a suitable anisotropic Sobolev space, and it furnishes quasi-optimal error estimates.