ti 11.11.08

Symmetrisen matriisin diagonalisointi

orthsymm.pdf (Pitäydytään tässä reaalisissa matriiseissa)
LAY 7.1 : Diagonalization of symmetric matr.
KRE[9] 8.4 : Eigenbases, diagonalization, Quadr. forms

Symmetrinen matriisi : $A^T = A$

Esim
$$A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

$$p(\lambda) = \begin{vmatrix} 6-\lambda & -2 & -1 \\ -2 & 6-\lambda & -1 \\ -1 & -1 & 5-\lambda \end{vmatrix} = \dots -(\lambda-8)(\lambda-6)(\lambda-3)$$
(kuin ihmeen kautta ?)

Rutiinilaskut :

$\lambda_1 = 8$, $\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\lambda_2 = 6$, $\vec{v}_2 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

$\lambda_3 = 3$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ LRT, mutta ei siinä kaikki !

$$\begin{cases} \vec{v}_1 \cdot \vec{v}_2 = v_1^T v_2 = 1 - 1 + 0 = 0 \\ \vec{v}_1 \cdot \vec{v}_3 = -1 + 1 + 0 = 0 \\ \vec{v}_2 \cdot \vec{v}_3 = -1 - 1 + 2 = 0 \end{cases}$$

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ on jopa <u>ortogonaalinen</u> joukko

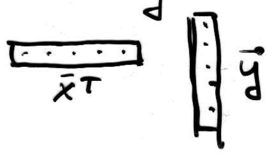
Normeeraus $|\vec{v}_1| = \sqrt{2}$, $|\vec{v}_2| = \sqrt{6}$, $|\vec{v}_3| = \sqrt{3}$

(Useim. merk. myös : $\|\vec{v}_1\|$, ...)

$\mathbb{R}^3$:n ortonormaali kanta :

$$\vec{u}_1 = \frac{1}{\sqrt{2}}\vec{v}_1 , \quad \vec{u}_2 = \frac{1}{\sqrt{6}}\vec{v}_2 , \quad \vec{u}_3 = \frac{1}{\sqrt{3}}\vec{v}_3$$

Muista $\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = \bar{x}^T \bar{y} = \langle \vec{x}, \vec{y} \rangle$



$$(A\vec{x}_1) \cdot \vec{x}_2 = (A\vec{x}_1 \cdot \vec{x}_2)^T = (\bar{x}_2^T A \bar{x}_1)^T$$

[(1×1) - matr.]

$$= (A\bar{x}_1)^T \bar{x}_2 = \bar{x}_1^T A^T \bar{x}_2 = \bar{x}_1 \cdot A^T \bar{x}_2$$

$$\langle A\vec{x}_1, \vec{x}_2 \rangle = \langle \vec{x}_1, A^T \vec{x}_2 \rangle$$

LAUSE Jos A on symmetrinen, niin eri ominaisavaruuksiin kuuluvat vektorit ovat ortogonaaliset.

Tod: Olk. $A\vec{x}_1 = \lambda_1 \vec{x}_1$, $A\vec{x}_2 = \lambda_2 \vec{x}_2$

$\lambda_1 \neq \lambda_2$

$$A\vec{x}_1 \cdot \vec{x}_2 = \lambda_1 \vec{x}_1 \cdot \vec{x}_2$$

$A^T = A$ ||

$$\vec{x}_1 \cdot A\vec{x}_2 = \vec{x}_1 \cdot \lambda_2 \vec{x}_2 = \lambda_2 \vec{x}_1 \cdot \vec{x}_2$$

$$\Rightarrow \underbrace{(\lambda_1 - \lambda_2)}_{\neq 0}(\vec{x}_1 \cdot \vec{x}_2) = 0 \quad \Rightarrow \vec{x}_1 \cdot \vec{x}_2 = 0,$$

eli $\vec{x}_1 \perp \vec{x}_2$. $\square$