Kompleksiset ominaisarvot

$\lambda = \alpha + i\beta \quad , \quad \bar{\lambda} = \alpha - i\beta$

$\vec{w} = \vec{u} + i\vec{v} \quad , \quad \bar{\vec{w}} = \vec{u} - i\vec{v}$

$\vec{y}_1(t) = e^{\lambda t}\vec{w} \quad , \quad \vec{y}_2(t) = e^{\bar{\lambda} t}\bar{\vec{w}}$

Myös $\frac{1}{2}(\vec{y}_1(t) + \vec{y}_2(t)) = \mathrm{Re}\, \vec{y}_1(t)$ ja

$\frac{1}{2i}(\vec{y}_1(t) - \vec{y}_2(t)) = \mathrm{Im}\, \vec{y}_1(t)$

ovat LRT ratk.

$$e^{\lambda t}\vec{w} = e^{\alpha t}(\cos\beta t + i\sin\beta t)(\vec{u} + i\vec{v})$$

$$= e^{\alpha t}\left[(\cos\beta t)\vec{u} - (\sin\beta t)\vec{v} + i((\sin\beta t)\vec{u} + (\cos\beta t)\vec{v})\right]$$

$$\vec{z}_1(t) = \mathrm{Re}(e^{\lambda t}\vec{w}) = e^{\alpha t}\left[(\cos\beta t)\vec{u} - (\sin\beta t)\vec{v}\right]$$

$$\vec{z}_2(t) = \mathrm{Im}(e^{\lambda t}\vec{w}) = e^{\alpha t}\left[\sin\beta t\,\vec{u} + (\cos\beta t)\vec{v}\right]$$

$$c_1\vec{z}_1(t) + c_2\vec{z}_2(t) =$$

$$e^{\alpha t}\left[(c_1\cos\beta t + c_2\sin\beta t)\vec{u} + (-c_1\sin\beta t + c_2\cos\beta t)\vec{v}\right]$$

$$= e^{\alpha t}\left[\vec{u} \mid \vec{v}\right]\begin{bmatrix}\cos\beta t & \sin\beta t \\ -\sin\beta t & \cos\beta t\end{bmatrix}\begin{bmatrix}c_1 \\ c_2\end{bmatrix}$$