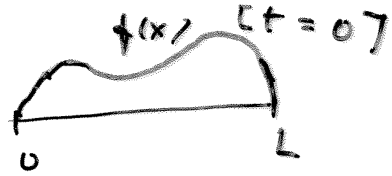
Lämpöyhtälö

$$\frac{\partial u}{\partial t} = c^2 \Delta u \qquad (\Delta = \nabla^2)$$

1-ulott: $\quad \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$



1) O-RE:t

Ominaisarvot: $\lambda_n = \frac{cn\pi}{L}$

Ominaisfunktiot: $u_n(x,t) = \sin\frac{n\pi x}{L} e^{-\lambda_n^2 t}$

AE toteutuu valitsemalla

$$u(x,t) = \sum_{n=1}^{\infty} b_n u_n(x,t),$$

missä $b_n = \frac{2}{L}\int_0^L f(x) \sin\frac{n\pi x}{L}\, dx$ — f:n sin-sarjan kertoimet

2) Eristetyt päät $(u_x(0,t) = u_x(L,t) = 0)$ "Neumann"

$$\lambda_n = \frac{cn\pi}{L}, \quad n = 0, 1, 2, \ldots$$

$$u_n(x,t) = \cos\frac{n\pi x}{L} e^{-\lambda_n^2 t},$$

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} e^{-\lambda_n^2 t}$$

$$a_0 = \frac{1}{L}\int_0^L f(x)\, dx, \quad a_n = \frac{2}{L}\int_0^L f(x) \cos\frac{n\pi x}{L}\, dx$$

} f:n cos-sarjan kertoimet