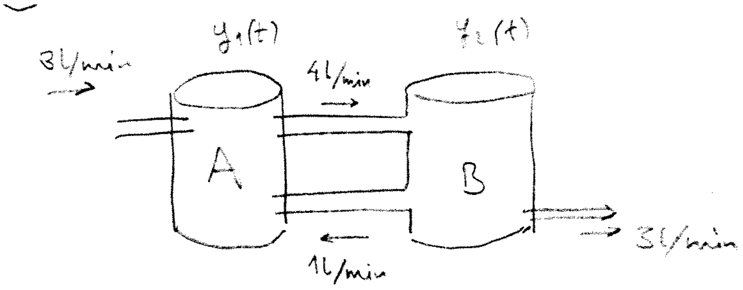


9LV

$V_A = 50\,l$ $\quad V_B = 50\,l$

y = suolan määrä A: $y_1(0) = 25\,kg$

B: $y_2(0) = 0$

$$y_1'(t) = \frac{1}{50} y_2(t) - \frac{4}{50} y_1(t)$$

$$y_2'(t) = \frac{4}{50} y_1(t) - \frac{1}{50} y_2(t) - \frac{3}{50} y_2(t)$$
$$= \frac{4}{50} y_1(t) - \frac{4}{50} y_2(t)$$

(1) $\bar{y}' = A\bar{y}$ $\qquad A = \begin{bmatrix} -\frac{4}{50} & \frac{1}{50} \\ \frac{4}{50} & -\frac{4}{50} \end{bmatrix}$

$$\bar{y} = \bar{x} e^{\lambda t}$$
$$\bar{y}' = \lambda \bar{x} e^{\lambda t} \overset{(1)}{=} A \bar{x} e^{\lambda t}$$
$$A\bar{x} = \lambda \bar{x}$$
$$(A - \lambda I)\bar{x} = 0$$

ominaisarvot:

$$\det(A - \lambda I) = \begin{bmatrix} -\frac{4}{50} - \lambda & \frac{1}{50} \\ \frac{4}{50} & -\frac{4}{50} - \lambda \end{bmatrix} = 0$$

$$\frac{16}{250} + \frac{8}{50}\lambda + \lambda^2 - \frac{4}{250} = 0$$
$$\lambda^2 + \frac{8}{50}\lambda + \frac{12}{250} = 0$$

$$\lambda = \frac{-\frac{8}{50} \pm \sqrt{\frac{64}{250} - 4 \cdot \frac{12}{250}}}{2} = -\frac{4}{50} \pm \frac{2}{50}$$

$$\lambda_1 = -\frac{2}{50} \quad \vee \quad \lambda_2 = -\frac{6}{50}$$

vektorit:

$\lambda_1 = -\frac{2}{50}$

$$\begin{bmatrix} -\frac{4}{50} + \frac{2}{50} & \frac{1}{50} \\ \frac{4}{50} & -\frac{2}{50} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0}$$

$$\begin{aligned} -2x_1 + x_2 &= 0 \\ 4x_1 - 2x_2 &= 0 \end{aligned} \Rightarrow \begin{aligned} x_1 &= 1 \\ x_2 &= 2 \end{aligned} \Rightarrow \bar{x}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\lambda_2 = -\frac{6}{50}$

$$\begin{bmatrix} \frac{2}{50} & \frac{1}{50} \\ \frac{4}{50} & \frac{2}{50} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0}$$

$$\begin{aligned} 2x_1 + x_2 &= 0 \\ 4x_1 + 2x_2 &= 0 \end{aligned} \Rightarrow \begin{aligned} x_1 &= 1 \\ x_2 &= -2 \end{aligned}$$

$$\Rightarrow \bar{x}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

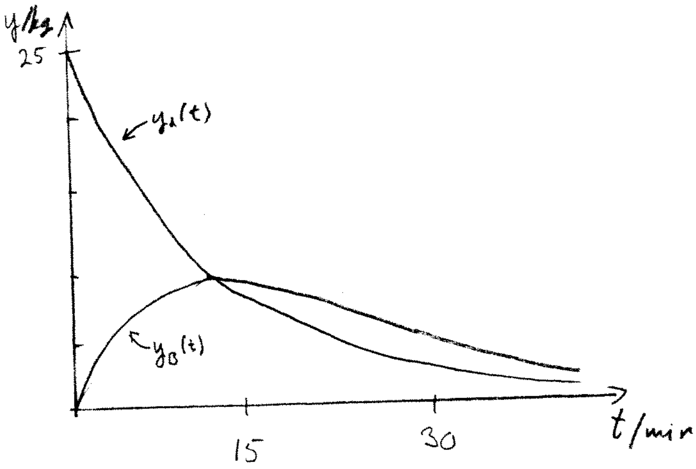
yl. ratkaisu

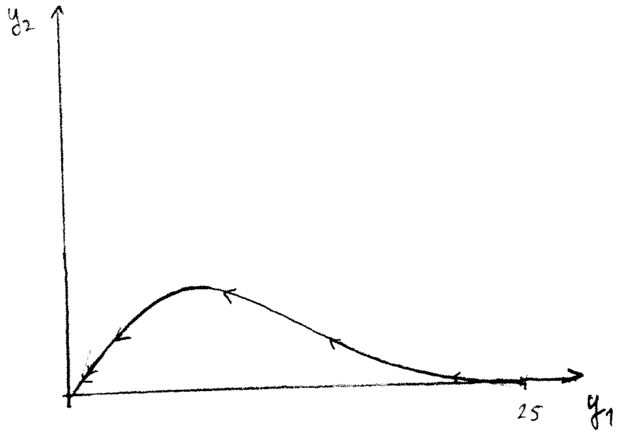
$$\bar{y} = c_1 \bar{x}_1 e^{\lambda_1 t} + c_2 \bar{x}_2 e^{\lambda_2 t} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-\frac{2}{50}t} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-\frac{6}{50}t}$$

alkuehdot: $\bar{y}(0) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \end{bmatrix}$ $\Leftrightarrow \begin{cases} c_1 + c_2 = 25 \\ 2c_1 - 2c_2 = 0 \end{cases} \Leftrightarrow c_1 = c_2 = \frac{25}{2}$

$$\bar{y} = \begin{bmatrix} 12{,}5 \\ 25 \end{bmatrix} e^{-\frac{2}{50}t} + \begin{bmatrix} 12{,}5 \\ -25 \end{bmatrix} e^{-\frac{6}{50}t}$$

$$\begin{cases} y_1 = 12{,}5\, e^{-\frac{2}{50}t} + 12{,}5\, e^{-\frac{6}{50}t} \\ y_2 = 25\, e^{-\frac{2}{50}t} - 25\, e^{-\frac{6}{50}t} \end{cases}$$





2.

(1) $\bar{y}' = A\bar{y}$ $\qquad \bar{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

ominaisarvot:

$$\begin{vmatrix} -\frac{3}{2}-\lambda & \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{9}{4} + 3\lambda + \lambda^2\right) - \frac{1}{4} = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = \frac{-3 \pm \sqrt{9 - 4 \cdot (+2)}}{2} = \frac{-3 \pm 1}{2}$$

$$\lambda_1 = -1 \qquad \lambda_2 = -2$$

ratkaisu: $\bar{y} = \bar{x} e^{\lambda t}$ $\qquad \bar{y}' = \lambda \bar{x} e^{\lambda t}$

(1) $\Rightarrow$ $\lambda \bar{x} e^{\lambda t} = A\bar{x} e^{\lambda t}$ $\qquad | : e^{\lambda t}$

$$A\bar{x} = \lambda \bar{x}$$

$$(A - \lambda I)\bar{x} = \bar{0}$$

om. vektorit:

$\lambda_1 = -1$ $\quad \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -2$ $\quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

yl. ratkaisu: $\quad \bar{y} = c_1 \bar{x}_1 e^{\lambda_1 t} + c_2 \bar{x}_2 e^{\lambda_2 t} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t}$

alkuarvot: $\quad \bar{y}(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \iff \begin{cases} c_1 + c_2 = 1 \\ c_1 - c_2 = 0 \end{cases} \iff c_1 = c_2 = \frac{1}{2}$

$$\Rightarrow \quad \underline{\underline{\bar{y} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} e^{-t} + \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} e^{-2t} \iff \begin{cases} y_1 = \frac{1}{2} e^{-t} + \frac{1}{2} e^{-2t} \\ y_2 = \frac{1}{2} e^{-t} - \frac{1}{2} e^{-2t} \end{cases}}}$$

kuvat liitteenä

3.

$\bar{y}' = A\bar{y}$ $\quad A = \begin{bmatrix} 1 & -1 \\ 9 & -5 \end{bmatrix}$ $\quad y(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\bar{y} = \bar{x}e^{\lambda t}$:

$\bar{y}' = \bar{x}\lambda e^{\lambda t}$ $\quad \Rightarrow \quad y' = \lambda\bar{x}e^{\lambda t} = A\bar{x}e^{\lambda t} \quad |: e^{\lambda t}$

$$A\bar{x} = \lambda\bar{x}$$

$$(A - \lambda I)\bar{x} = \bar{0}$$

om. arvot

$$\begin{vmatrix} 1-\lambda & -1 \\ 9 & -5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-5-\lambda) + 9 = 0$$

$$-5 + 4\lambda + \lambda^2 + 9 = 0$$

$$\lambda^2 + 4\lambda + 4 = 0$$

$$(\lambda+2)^2 = 0$$

$$\lambda = -2$$

om. vektorit:

$$\begin{bmatrix} 1+2 & -1 \\ 9 & -5+2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0}$$

$$\begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0}$$

$$\begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

ratkaisu 1:

$$\bar{y}_I = \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-2t}$$

ratkaisu 2:

yrite: $\bar{y} = \bar{x}te^{\lambda t} + \bar{u}e^{\lambda t}$

$$\bar{y}' = \bar{x}e^{\lambda t} + \bar{x}\lambda te^{\lambda t} + \bar{u}\lambda e^{\lambda t} = A\bar{y}$$

$$\bar{x}e^{\lambda t} + \bar{x}\lambda te^{\lambda t} + \bar{u}\lambda e^{\lambda t} = A\bar{x}te^{\lambda t} + A\bar{u}e^{\lambda t} \quad |: e^{\lambda t}$$

$$\bar{x} + \bar{x}\lambda t + \bar{u}\lambda = A\bar{x}t + A\bar{u} \quad | A\bar{x} = \lambda\bar{x}$$

$$\bar{x} + \cancel{\bar{x}\lambda t} + \bar{u}\lambda = \cancel{\lambda\bar{x}t} + A\bar{u}$$

$$A\bar{u} = \bar{x} + \lambda\bar{u}$$

$$(A - \lambda I)\bar{u} = \bar{x}$$

$\bar{u}$:

$$\begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Leftrightarrow \begin{cases} 3u_1 - u_2 = 1 \\ 9u_1 - 3u_2 = 3 \end{cases} \Leftrightarrow \bar{u} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\bar{y}_{II} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} te^{-2t} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-2t}$$

$$\bar{y} = c_1\bar{y}_I + c_2\bar{y}_{II} = c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-2t} + c_2\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} t + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) e^{-2t}$$

$$\bar{y}(0) = c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}0\\-1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$

$c_1 = 2$

$3c_1 - c_2 = 3$

$c_2 = 6-3 = 3$

$$\Rightarrow \quad \bar{y} = \begin{bmatrix}2\\6\end{bmatrix}e^{-2t} + \begin{bmatrix}3\\9\end{bmatrix}te^{-2t} + \begin{bmatrix}0\\-3\end{bmatrix}e^{-2t} = \begin{bmatrix}2\\3\end{bmatrix}e^{-2t} + \begin{bmatrix}3\\9\end{bmatrix}te^{-2t}$$

kuvaaja liitteenä

4.

$$x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$$

merk. $\begin{cases} y_1 = x \\ y_2 = x' \end{cases}$

$y_2' + \frac{c}{m}y_2 + \frac{k}{m}y_1 = 0$

$y_2' = -\frac{c}{m}y_2 - \frac{k}{m}y_1$

$$\boxed{\begin{aligned} \det A &= \frac{k}{m} = \alpha^2 \\ \operatorname{tr} A &= -\frac{c}{m} = \beta \end{aligned}}$$

$$\bar{y} = \begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}x\\x'\end{bmatrix} \qquad \bar{y}' = \begin{bmatrix}y_1'\\y_2'\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{c}{m}\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}$$

$$\bar{y}' = \begin{bmatrix}y_1'\\y_2'\end{bmatrix} = \begin{bmatrix}x'\\x''\end{bmatrix} \qquad \bar{y}' = A\bar{y} \qquad \bar{y} = \bar{x}e^{\lambda t}$$

$$\bar{y}' = \bar{x}\lambda e^{\lambda t} = A\bar{x}e^{\lambda t} \qquad \| : e^{\lambda t}$$

$$\lambda\bar{x} = A\bar{x}$$

$$(A - \lambda I)\bar{x} = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -\frac{k}{m} & -\frac{c}{m} - \lambda \end{vmatrix} = 0 \quad \Leftrightarrow \quad \begin{aligned} &\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0 \\ &\lambda^2 - \beta\lambda + \alpha^2 = 0 \end{aligned}$$

$$\lambda = \frac{-\frac{c}{m} \pm \sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}}{2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= \frac{1}{2}\beta \pm \sqrt{(\tfrac{1}{2}\beta)^2 - \alpha^2}$$

a) ei vaimennusta $c = 0$

$\lambda = \pm\sqrt{-\frac{k}{m}} = \pm i\sqrt{\frac{k}{m}} = \pm i\alpha$ imag. om. arvot, $\alpha = \sqrt{\frac{k}{m}} = \sqrt{\det A}$

$$\begin{bmatrix} -i\sqrt{\frac{k}{m}} & 1 \\ -\frac{k}{m} & -i\sqrt{\frac{k}{m}} \end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \bar{0} \qquad \begin{aligned} -i\sqrt{\tfrac{k}{m}}x_1 + x_2 &= 0 \\ -\tfrac{k}{m}x_1 - i\sqrt{\tfrac{k}{m}}x_2 &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1 &= 1 \\ x_2 &= i\sqrt{\tfrac{k}{m}} = i\alpha \end{aligned} \qquad \bar{x}_1 = \begin{bmatrix}1\\ i\sqrt{\frac{k}{m}}\end{bmatrix} = \begin{bmatrix}1\\ i\alpha\end{bmatrix}$$

$$\begin{bmatrix} i\sqrt{\frac{k}{m}} & 1 \\ -\frac{k}{m} & i\sqrt{\frac{k}{m}} \end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x}_2 = \begin{bmatrix}1\\ -i\sqrt{\frac{k}{m}}\end{bmatrix} = \begin{bmatrix}1\\ -i\alpha\end{bmatrix}$$

$$\bar{y} = c_1\bar{x}_1e^{\lambda t} + c_2\bar{x}_2e^{\lambda_2 t} = c_1\begin{bmatrix}1\\-i\alpha\end{bmatrix}e^{i\alpha t} + c_2\begin{bmatrix}1\\i\alpha\end{bmatrix}e^{-i\alpha t}$$

$\Rightarrow$ värähtelevä ratkaisu (ks. kuva 4a liitteen[ä])

$$= c_1\begin{bmatrix}\cos(\alpha t) + i\sin(\alpha t) \\ -i\alpha\cos(\alpha t) + \alpha\sin(\alpha t)\end{bmatrix} + c_2\begin{bmatrix}\cos(\alpha t) - i\sin(\alpha t) \\ i\alpha\cos(\alpha t) + \alpha\sin(\alpha t)\end{bmatrix}$$

b) alivaimennus $c^2 < 4mk$

$D < 0$

$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} < 0$

$c^2 < \frac{4m^2k}{m}$

$c^2 < 4mk \Rightarrow$ imag. juuret

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} = -\frac{c}{2m} \pm i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\lambda_1 = -\frac{c}{2m} + i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\begin{bmatrix} \frac{c}{2m} - i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} & 1 \\ -\frac{k}{m} & -\frac{c}{2m} - i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \qquad \bar{x}_1 = \begin{bmatrix} 1 \\ -\frac{c}{2m} - \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \end{bmatrix}$$

$$\lambda_2 = -\frac{c}{2m} - \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\begin{bmatrix} \frac{c}{2m} + i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} & 1 \\ -\frac{k}{m} & -\frac{c}{2m} + i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \qquad \bar{x}_2 = \begin{bmatrix} 1 \\ \frac{c}{2m} - i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \end{bmatrix}$$

$$\bar{y} = c_1 \bar{x}_1 e^{\lambda_1 t} + c_2 \bar{x}_2 e^{\lambda_2 t} = c_1 \bar{x}_1 e^{\left(-\frac{c}{2m} + i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}\right)t} + c_2 \bar{x}_2 e^{\left(\frac{c}{2m} - i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}\right)}$$

$$= c_1 \begin{bmatrix} 1 \\ \frac{1}{2}\beta - i\sqrt{\alpha^2 - \left(\frac{1}{2}\beta\right)^2} \end{bmatrix} e^{\left(\frac{1}{2}\beta + i\sqrt{\alpha^2 - \left(\frac{1}{2}\beta\right)^2}\right)t} + c_2 \begin{bmatrix} 1 \\ \frac{1}{2}\beta - i\sqrt{\alpha^2 - \left(\frac{1}{2}\beta\right)^2} \end{bmatrix} e^{\left(\frac{1}{2}\beta - i\sqrt{\alpha^2 - \left(\frac{1}{2}\beta\right)^2}\right)t}$$

$\Rightarrow$ vaimennettu värähtely

(ks. kuva 4b liitteenä)

c) $c^2 = 4mk$

$$\lambda = -\frac{\sqrt{4mk}}{2m} \pm \sqrt{\frac{4mk}{4m^2} - \frac{k}{m}} = -\sqrt{\frac{k}{m}} = -\alpha \quad \text{kaksinkertainen juuri}$$

$$\begin{bmatrix} \sqrt{\frac{k}{m}} & 1 \\ -\frac{k}{m} & -\sqrt{\frac{k}{m}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \qquad \bar{x} = \begin{bmatrix} 1 \\ -\sqrt{\frac{k}{m}} \end{bmatrix}$$

yrite: $\bar{y}_{II} = \bar{x}te^{\lambda t} + \bar{u}e^{\lambda t}$

$\bar{y}' = \bar{x}e^{\lambda t} + \bar{x}t\lambda e^{\lambda t} + \bar{u}\lambda e^{\lambda t} = A\bar{y}$

$\bar{x}e^{\lambda t} + \bar{x}t\lambda e^{\lambda t} + \bar{u}\lambda e^{\lambda t} = A\bar{x}te^{\lambda t} + A\bar{u}e^{\lambda t}$

$\bar{x} + \bar{x}t\lambda + \bar{u}\lambda = A\bar{x}t + A\bar{u}$

$\bar{x} + \cancel{\lambda\bar{x}t} + \lambda\bar{u} = \cancel{\lambda\bar{x}t} + A\bar{u}$

$(A - \lambda I)\bar{u} = \bar{x}$

$$\begin{bmatrix} \sqrt{\frac{k}{m}} & 1 \\ -\frac{k}{m} & -\sqrt{\frac{k}{m}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -\sqrt{\frac{k}{m}} \end{bmatrix} \qquad \bar{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bar{y} = c_1 \bar{x} e^{\lambda t} + c_2\left(\bar{x}te^{\lambda t} + \bar{u}e^{\lambda t}\right)$$

$$= c_1 \begin{bmatrix} 1 \\ -\sqrt{\frac{k}{m}} \end{bmatrix} e^{\lambda t} + c_2\left(\begin{bmatrix} 1 \\ -\sqrt{\frac{k}{m}} \end{bmatrix} te^{\lambda t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{\lambda t}\right)$$

degeneroitunut noodi (sillä $\lambda < 0$)

$\Rightarrow$ stabiili, nielu

(ks. kuva 4c liitteenä)

d) ylivaimennus $c^2 > 4mk$

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\lambda_1 = \frac{1}{2}\beta + \sqrt{(\tfrac{1}{2}\beta)^2 - \alpha^2} \qquad \lambda_2 = \frac{1}{2}\beta - \sqrt{(\tfrac{1}{2}\beta)^2 - \alpha^2}$$

$D = \left(\frac{c}{2m}\right)^2 - \frac{k}{m} > 0$

$c^2 > 4mk$ ok.

$\Rightarrow$ kaksi reaalista ominaisarvoa

λ_1:
$$\begin{bmatrix} -\frac{1}{2}\beta - \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} & 1 \\ -\alpha^2 & \frac{1}{2}\beta - \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x}_1 = \begin{bmatrix} 1 \\ \frac{1}{2}\beta + \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix}$$

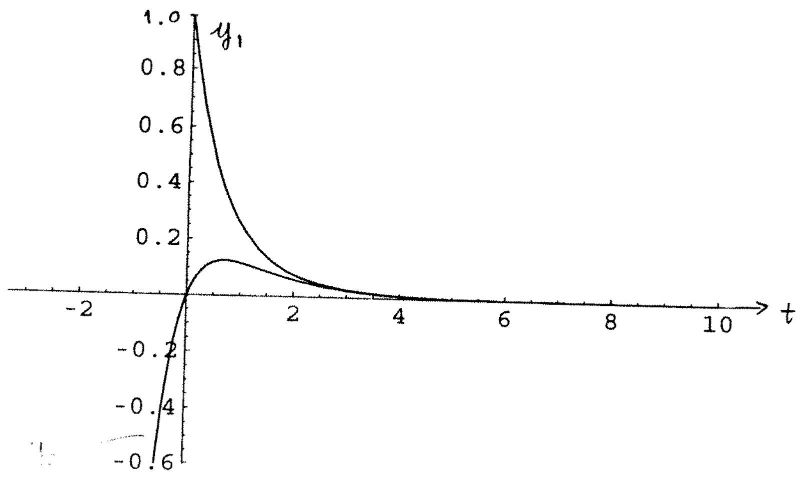
λ_2:
$$\begin{bmatrix} -\frac{1}{2}\beta + \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} & 1 \\ -\alpha^2 & \frac{1}{2}\beta + \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{0} \quad \Rightarrow \quad \bar{x}_2 = \begin{bmatrix} 1 \\ \frac{1}{2}\beta - \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix}$$

ratk.
$$\bar{y} = c_1 \bar{x}_1 e^{\lambda_1 t} + c_2 \bar{x}_2 e^{\lambda_2 t}$$
$$= c_1 \begin{bmatrix} 1 \\ \frac{1}{2}\beta + \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix} e^{\frac{1}{2}\beta + \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2}} + c_2 \begin{bmatrix} 1 \\ \frac{1}{2}\beta - \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2} \end{bmatrix} e^{\frac{1}{2}\beta - \sqrt{(\frac{1}{2}\beta)^2 - \alpha^2}}$$

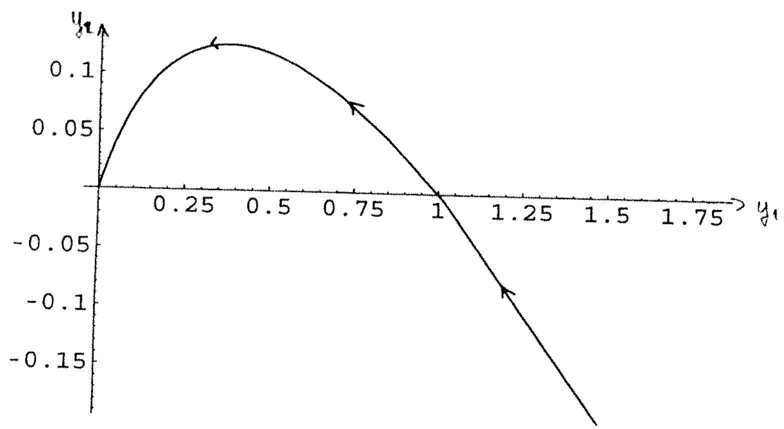
$\Rightarrow$ vaimeneva ratkaisu

(ks. kuva 4d liitteenä)

2.

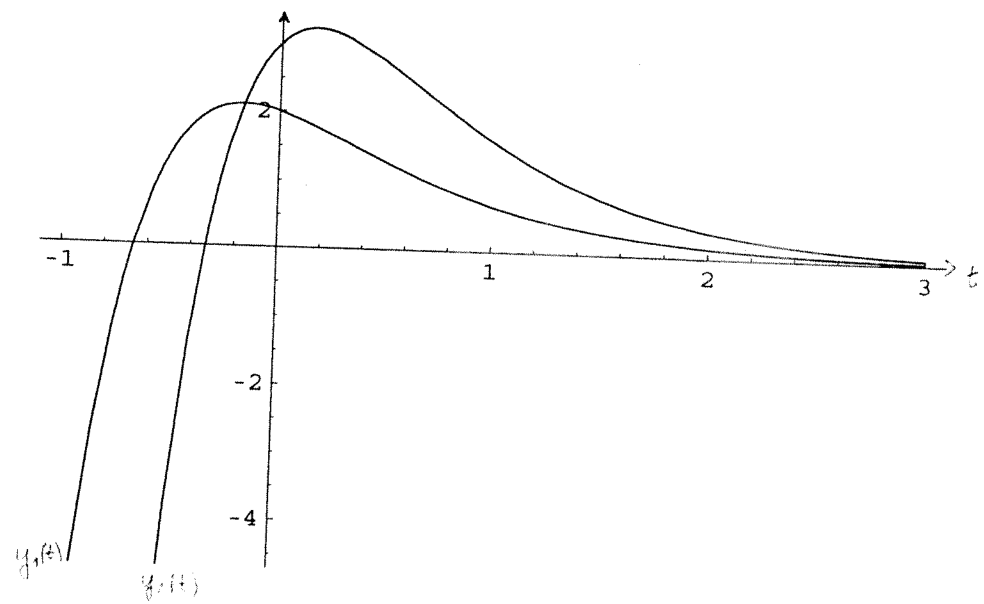


1.0
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-2
2
4
6
8
10
t
y_1



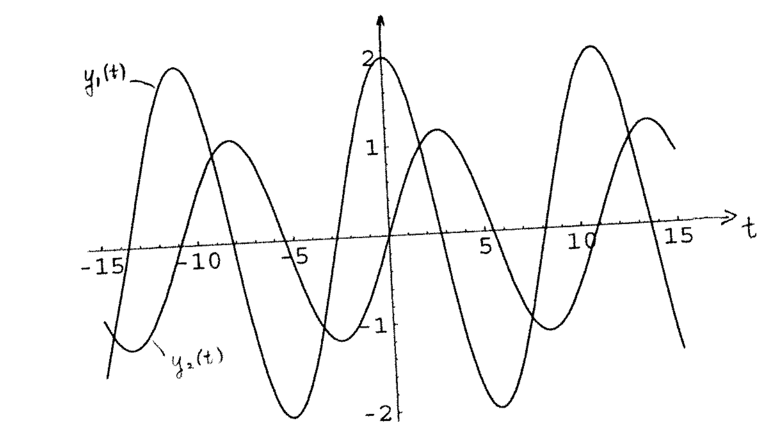
y_2
0.1
0.05
-0.05
-0.1
-0.15
0.25
0.5
0.75
1
1.25
1.5
1.75
y_1

3.

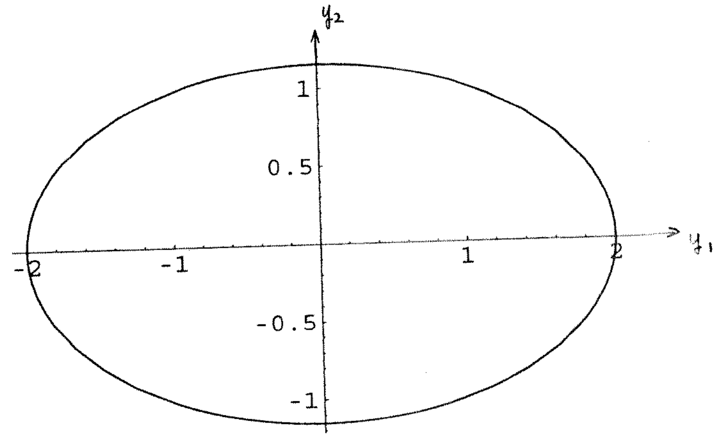


2
-2
-4
-1
1
2
3
t
$y_1(t)$

4a)

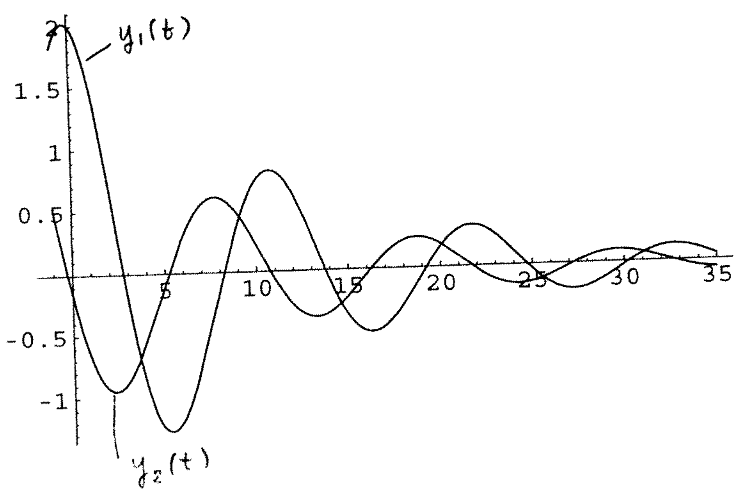


y₁(t)
y₂(t)
t
2
1
-1
-2
-15
-10
-5
5
10
15

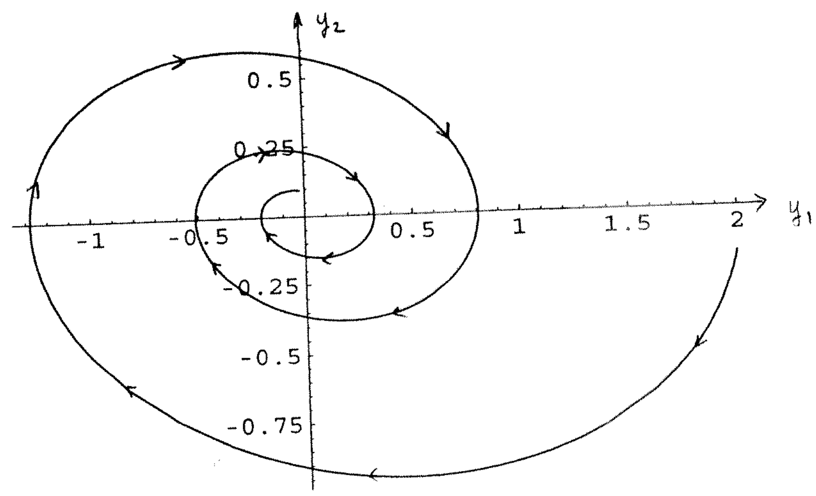


y₂
y₁
1
0.5
-0.5
-1
-2
-1
1
2

4 b)

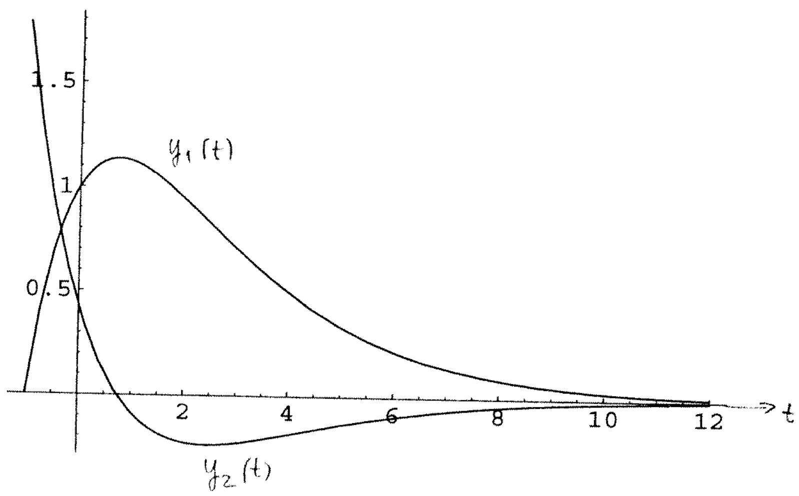


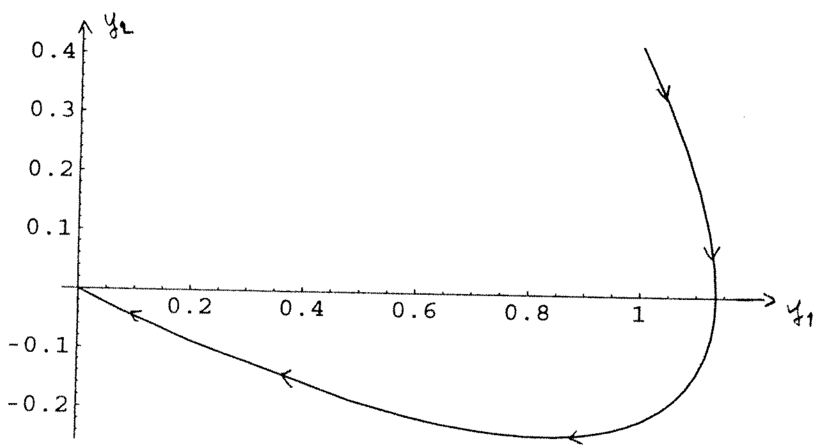
y₁(t)
y₂(t)
2
1.5
1
0.5
-0.5
-1
5
10
15
20
25
30
35



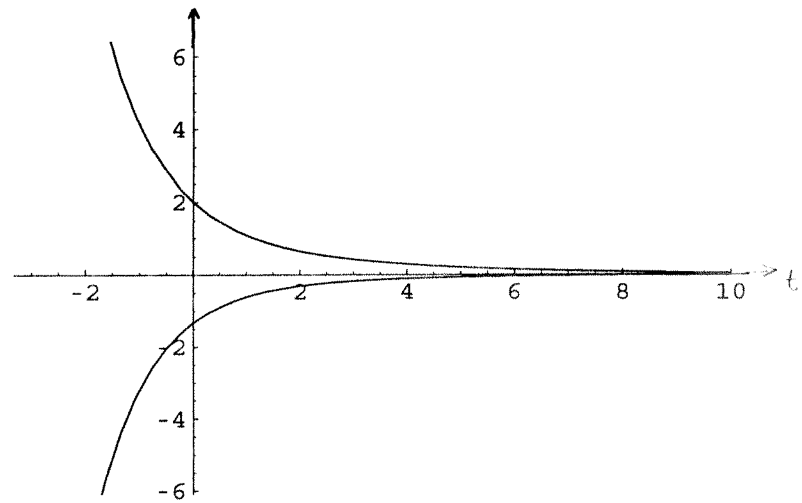
y₂
y₁
0.5
0.25
-0.25
-0.5
-0.75
-1
-0.5
0.5
1
1.5
2

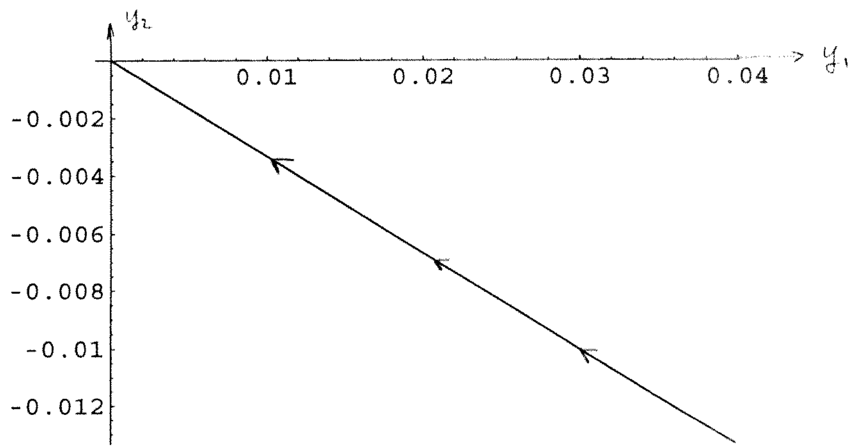
4c)


1.5
1
0.5
2
4
6
8
10
12
t
y1(t)
y2(t)


y2
0.4
0.3
0.2
0.1
-0.1
-0.2
0.2
0.4
0.6
0.8
1
y1

4d)


6
4
2
-2
-4
-6
-2
2
4
6
8
10
t


y2
0.01
0.02
0.03
0.04
y1
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012