Mat-5.3741 Theory of Elasticity (5 cp) L **Spring 2007**
Stenberg/Juntunen

Exercise 6

Problem 1
Let the stress tensor be given in the system of principal axis

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}.$$

Compute maximum value of the shear force and its direction. (Note: You will get three alternatives of which you choose the biggest.)

Problem 2
Consider the general constitutive law

$$\sigma_{ij} = \sum_{k,l=1}^{3} C_{ijkl}\epsilon_{kl}$$

with the relations

$$\begin{aligned} C_{ijkl} &= C_{klij} \\ C_{ijkl} &= C_{ijlk} \\ C_{ijkl} &= C_{jikl} \end{aligned}$$

How many independent constants are there in C_{ijkl}?

Problem 3 **(home exercise)**
Find the expression for the biharmonic operator Δ^2 in the polar coordinates (r, φ).
Proove that Airy's function

$$\Phi(r,\varphi) = \frac{\sigma_0}{4}\left[r^2 - 2a^2 \ln r - \frac{(r^2-a^2)^2}{r^2}\cos 2\varphi\right]$$

satisfies the biharmonic equation

$$\Delta^2 \Phi = 0.$$

What are the stress components τ_{rr}, $\tau_{r\varphi}$ and $\tau_{\varphi\varphi}$? Proove that

- on the circle $r = a$ the free boundary condition $\underline{\underline{\tau}}\,\underline{n} = \underline{0}$ is satisfied
- it holds true the limit

$$\lim_{x\to\pm\infty} \underline{\underline{\tau}}(x,y) = \begin{pmatrix} \sigma_0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Draw the stress components on coordinate-axes ($x = 0$, $|y| \geq a$ and $y = 0$, $|x| \geq a$). The problem is related to the situation shown in figure below.