Helsinki University of Technology
Mat-5.156 Theory of Elasticity **Stenberg / Tuominen**
8. exercise on Friday 8th April 2005

Note that the exercise session will be held in the place of Friday's lecture.

Please return also the answers to the project exercise (ex. 4.5).

1. (Diaz-Weinstein equation, Sokolnikoff, 2nd Ed. p.177) In the Saint-Venant theory of torsion holds
$$\frac{M}{G\alpha} = J + \int_R \left(x\frac{\partial \phi}{\partial y} - y\frac{\partial \phi}{\partial x} \right) dA,$$
where J is the polar moment of inertia of the cross section R:
$$J = \int_R (x^2 + y^2)\, dA.$$
Prove that
$$\frac{M}{G\alpha} = J - \int_R \left((\frac{\partial \phi}{\partial x})^2 + (\frac{\partial \phi}{\partial y})^2 \right) dA.$$
Conclusion?

2. Consider $\Omega = \{x^2 + y^2 \leq R^2\}$ as the domain of Kirchhoff plate and uniform load $= 1/(\pi a^2)$ in domain $\{x^2 + y^2 \leq a^2\}$.
Solve the problem with Mathematica/Maple both for fixed and simply supported boundary. Draw all quantities, also in case $a \to 0$.

   Check also if your program is able to use Kelvin functions in the case of plate on elastic base.

   Joker. Is your symbolic program able to manage the circular Reissner-Mindlin plate.

3. (**Home exercise.**) Consider simply supported Kirchhoff plate in domain $\Omega = (0, 2a) \times (0, 2b)$ and uniform load $p(x, y) = \frac{1}{4cd}$ in domain $(a - c, a + c) \times (b - d, b + d)$. Solve the problem using Fourier series and draw the deflection with Mathematica/Maple. Calculate and draw also the expressions for moments and shear forces. Examine cases $(c, d) \to (0, 0)$ (point load) ja $(c, d) \to (a, b)$. What can you say about the convergence of the series of different quatities and in different cases?