Helsinki University of Technology
Mat-5.156 Theory of Elasticity **Stenberg / Tuominen**
7. exercise on Tuesday 22nd March 2005

1. Consider a beam for which the energy is given as

$$I(w) \quad = \quad \frac{1}{2}\int_0^L EI(w''(x))^2 dx - \int_0^L f(x)w(x)dx,$$

and minimization is done in sets

$$V_1 = \{\, v \,|\, \int_0^L EI(v''(x))^2 dx < \infty,\ v(0) = v(L) = 0,\ \}.$$

$$V_2 = \{\, v \,|\, \int_0^L EI(v''(x))^2 dx < \infty,\ v(0) = v(L) = 0, v'(0) = v'(L) = 0\ \}.$$

Write the solution by means of Green's functions.

2. Consider the Neumann problem: find u such that

$$-\Delta u = f \quad \text{in}\, \Omega$$
$$\nabla u \cdot \underline{n} = g \quad \text{on}\, \partial\Omega.$$

Show that the neccessary condition for the existence of the solution is

$$\int_\Omega f\, dx + \int_{\partial\Omega} g\, ds = 0.$$

In the torsion problem based on minimization of potential energy the following boundary value problem arises:

$$\Delta\phi = 0 \quad \text{in}\, R,$$
$$(\frac{\partial\phi}{\partial x} - y)n_x + (\frac{\partial\phi}{\partial y} + x)n_y = 0. \quad \text{on}\, \partial R.$$

Does a solution exist?

3. Function u is subharmonic if it satisfies condition

$$\Delta u \geq 0 \quad \text{in}\, R.$$

Theorem (Maximum principle, e.g. Evans. PDE's, Sec. 6.4): Subharmonic function attains its maximum on the boundary of domain.

Using this theorem proof that the shear force vector

$$\underline{T} = (\sigma_{xz}, \sigma_{yz}) = G\alpha\left(\frac{\partial\Psi}{\partial y}, -\frac{\partial\Psi}{\partial x}\right)$$

attains its maximum length on the boundary of the cross section (Ψ is the stress function.)

4. (**Home exercise**) Calculate the approximation for the torque

$$2\int_R \Psi\, dA$$

of retangular cross section $[-a, a] \times [-b, b]$ using Galerkin method with one basis function

$$\psi = \cos\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi y}{2b}\right).$$

Calculate more accurate value using Fourier-series (by hand or Maple/Mathematica).