**Helsinki University of Technology**
**Mat-5.156 Theory of Elasticity** **Stenberg / Tuominen**
**6. exercise on Tuesday 15th March 2005**

1. (**Home exercise**) Consider the Euler-Bernoulli beam with zero boundary conditions at one end, and moment $m$ and shear force $q$ given at the the other end:
$$\begin{aligned}(EIw''(x))'' &= f(x), \quad 0 < x < L\\ w(0) = w'(0) &= 0,\\ EIw''(L) + m &= 0,\\ (EIw''(L))' + q &= 0.\end{aligned}$$
Write the problem as minimization of the potential energy, minimization of the complementary energy (in terms of the moment) and as Hellinger-Reissner principle (deflection and moment).

2. Consider a beam for which the energy is
$$I(w) = \frac{1}{2}\int_0^L EI[w''(x)]^2 dx - Mw'(L/2) - Pw(L/3) - \int_0^L f(x)w(x)dx,$$
and minimization is done over the set
$$V = \{\, v \,|\, \int_0^L EI[v''(x)]^2 dx < \infty,\ v(0) = 0,\ v'(L) = 0\}.$$
Write the strong form of the problem, i.e the differential equation with boundary and compatibility conditions.

3. (Maple, Mathematica) What is the deflection curve of the simply supported ($w_T(0) = w_T(L) = 0$) Timoshenko beam loaded with linear force distribution $f(x) = q_0/Lx$? The cross section of the beam is retangular. Calculate the value of the ratio $w_T(L/2)/w(L/2)$, where $w_T$ and $w$ are the deflections according to the Timoshenko theory and ordinary theory of beams respectively. Assume that $\nu = 1/3$ and $h/L = 1/100$.

4. Using forms $\tau_{xx} = Mz/I$ and $\tau_{xz} = V/A$ show that Reissner functional for the Timoshenko beam is
$$I_R = \int_0^L [-M(w'' - \beta) + \beta V - \frac{M^2}{2EI} - \frac{V^2}{2kAG} - qw]dx,$$
where the shear factor $k$ is used in complementary energy. Show that extremizing Reissner functional the equilibrium equations for Timoshenko beam are attained as well as stress-strain relations. Show also that boundary conditions are satisfied.