Helsinki University of Technology
Mat-5.156 Theory of Elasticity **Stenberg / Tuominen**
4.exercise on Tuesday 1st March 2005

1. Proof that the variational problem

$$\int_0^1 (y'(x))^2\,dx \rightarrow \text{ extremum},$$

with boundary conditions $y(0) = y(1) = 0$ and constraint

$$\int_0^1 (y(x))^2\,dx = 1,$$

leads to the eigenvalue problem: find $y(x) \not\equiv 0$, $y(0) = y(1) = 0$ and λ such that

$$y''(x) = \lambda y(x), \quad 0 < x < 1.$$

2. Find a curve $y = y(x)$, $y(-a) = y(a) = 0$, such that the length of the curve on interval $[-a, a]$ is minimized and the area between the curve and the x-axis is constant A.

3. Constitutive model (=material model) of elastic material

$$\tau_{ij} = c_{ijkl}\varepsilon_{kl}(\underline{u}).$$

is positive definite, or there exists such $C > 0$ that

$$c_{ijkl}\xi_{ij}\xi_{kl} \geq C\xi_{ij}\xi_{ij}.$$

Proof that the inverse relation of the constitutive model written as a_{ijkl},

$$\varepsilon_{kl}(\underline{u}) = a_{ijkl}\tau_{ij},$$

is also positive definite. Show also that the equation

$$c_{ijkl}\varepsilon_{ij}(\underline{u})\varepsilon_{kl}(\underline{u}) = a_{ijkl}\tau_{ij}\tau_{kl}.$$

holds true.

4. (**Home exercise**). Consider the problem of beam in extension: find such u that

$$\begin{aligned} \frac{d}{dx}\Big(EA\frac{du(x)}{dx}\Big) + f(x) &= 0,\ 0 < x < L, \\ u(0) &= 0, \\ EAu'(L) + ku(L) &= 0. \end{aligned}$$

Write the problem both as a minimization problem and as a variational problem (principle of virtual work).

Which boundary value problem arises at limit $k \rightarrow \infty$? Proof this more precisely: Mark the energy norm with $|||\cdot|||_k$ (dependent on spring constant k). Let u_∞ be the candidate for the limit solution. Proof that ($u = u_k$)

$$|||u_\infty - u|||_k \leq k^{-1/2}|EAu'_\infty(L)|.$$

5. **Project for next three weeks**. Get acquainted with MATLAB's PDE TOOLBOX (`>>pdetool`). Solve the problem related to the exercise problem 3.4 using MATLAB. Credits for this problem equate two home exercises.