Helsinki University of Technology
Mat-5.156 Theory of Elasticity **Stenberg / Tuominen**
3. exercise on Tuesday 22nd February 2005

1. In the book it is claimed that the relations

$$\begin{aligned} C_{ijkl} &= C_{jikl}, \\ C_{ijkl} &= C_{ijlk}, \\ C_{ijkl} &= C_{klij} \end{aligned}$$

   imply that there exists at most 21 independent constants in C. Verify that.

2. (Chain line.) Consider a perfectly flexible string whose mass per unit length is ρ. The length of the string is L and it hangs between points (x_1, y_1) and (x_2, y_2). Find the curve of the string when it is known that the potential energy is minimized.

3. Let the curve $y(x)$, $y(x_1) = y_1$, $y(x_2) = y_2$, rotate around the x-axis. Find the curve for which the area of the surface of revolution is at minimum. You can use the Maple/Mathematica to solve the Euler equation.

4. (Home exercise.) Find the expression for the biharmonic operator Δ^2 in the polar coordinates (r, φ). Proof that Airy's function

$$\Phi(r,\varphi) = \frac{\sigma_0}{4}\left[r^2 - 2a^2 \ln r - \frac{(r^2 - a^2)^2}{r^2}\cos 2\varphi\right]$$

   satisfies the biharmonic equation

$$\Delta^2 \Phi = 0.$$

   What are the stress components τ_{rr}, $\tau_{r\varphi}$ and $\tau_{\varphi\varphi}$? Proof that

   - on the circle $r = a$ the free boundary condition $\underline{\underline{\tau}}\,\underline{n} = \underline{0}$ is satisfied
   - it holds true the limit

$$\lim_{x \to \pm\infty} \underline{\underline{\tau}}(x,y) = \begin{pmatrix} \sigma_0 & 0 \\ 0 & 0 \end{pmatrix}.$$

   Draw the stress components on coordinate-axes ($x = 0$, $|y| \geq a$ and $y = 0$, $|x| \geq a$). The problem is related to the situation shown in figure 1.

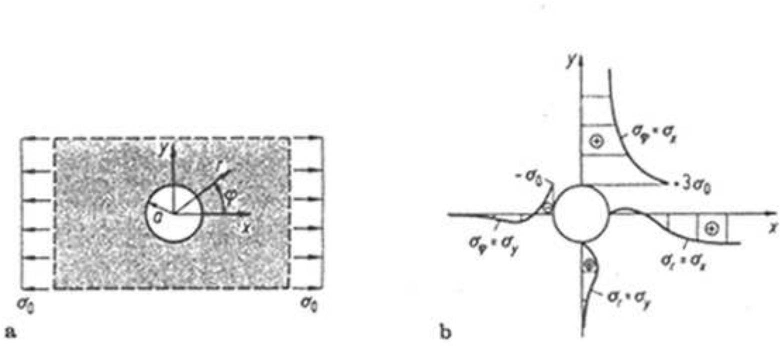

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Figure 1: problem 4