Helsinki University of Technology
Mat-5.156 Theory of Elasticity **Stenberg / Tuominen**
1. exercise on Tuesday 8th February 2005

1. Let $\Omega \subset \mathbb{R}^3$ be connected, bounded, with piecewise smooth boundary etc. and let $f : \Omega \to \mathbb{R}$ continuously differentiable. Explain with common sense that it holds

$$\int_\Omega \frac{\partial f}{\partial x_i}\, dV = \int_{\partial\Omega} f \underline{n} \cdot \underline{e}_i \, dS,$$

where $f = f(x_1, x_2, x_3)$, $\underline{n}$ is the outer normal vector of $\partial\Omega$ and $\underline{e}_i, i = 1, 2, 3$, are the basis vectors.

2. Let $\Omega \subset \mathbb{R}^3$ be as above and $f : \Omega \to \mathbb{R}$ continuous. Proof that if for every measurable subset $D \subset \Omega$ holds

$$\int_D f dV = 0,$$

then $f = 0$.

3. Write and derive Green's I identity $(u, v : \Omega \to \mathbb{R})$

$$\int_\Omega \nabla u \cdot \nabla v \, dV + \int_\Omega u \nabla^2 v \, dV = \int_{\partial\Omega} u \nabla v \cdot \underline{n} \, dS$$

using Einstein's notation for summation and differentiation.

4. (Home exercise.) Let $\underline{\underline{\sigma}}$ and $\underline{v}$ be tensor and vector valued functions.

   (a) Write the following equation using sum and partial differentiation notations:

$$\int_\Omega \sigma_{ij,j} v_i \, dV = -\int_\Omega \sigma_{ij} v_{i,j} \, dV + \int_{\partial\Omega} \sigma_{ij} n_j v_i \, dS$$

   (b) Derive the identity using Einstein's notations.