Mat-1.187 Dynaamisten systeemien erikoiskurssi

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3. Laskuharjoitus

pe 18.2. klo 14-16 U322

Huom. `Matlab`-ohjelmat joihin viitataan löytyvät kurssin www-sivuilta
http://www.math.hut.fi/teaching/dynsysek00/index.html.fi

1. (monisteen tehtävä 2.3.1). Consider $\boldsymbol{f}(\boldsymbol{x}) = 0$ and let $g : \mathbb{R}^n \to \mathbb{R}$ be defined by $g(\boldsymbol{x}) = \frac{1}{2}|\boldsymbol{f}(\boldsymbol{x})|^2$. Assuming invertibility of $D\boldsymbol{f}$, show that for a small enough h_k

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - h_k \left[D\boldsymbol{f}(\boldsymbol{x}^k)\right]^{-1} \boldsymbol{f}(\boldsymbol{x}^k),$$

   will lead to a decrease in $g(\boldsymbol{x})$. Infer that there are suitable step size choices for ultimately obtaining a solution. (Hint: show that the Newton direction is a *direction of descent*; that is, if we let $\boldsymbol{v} = [D\boldsymbol{f}(\boldsymbol{x})]^{-1} \boldsymbol{f}(\boldsymbol{x})$, then $\langle \boldsymbol{v}, \nabla g\rangle < 0$.)

2. (monisteen tehtävä 2.3.2). Modify the `Matlab` program `Newton.m` to perform damped Newton's method. A simple and effective strategy to find suitable step-sizes for damped Newton is by progressive halving:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - 1/2^j \left[D\boldsymbol{f}(\boldsymbol{x}^k)\right]^{-1} \boldsymbol{f}(\boldsymbol{x}^k), \quad j = 0, 1, \ldots,$$

   until a decrease in the norm of $\boldsymbol{f}$ is achieved (of course, in practice, we will damp up to a minimal value after which we will stop or retry with a different guess). Can you find a scalar problem where the use of damped Newton is beneficial? Note that in the damped Newton's method, even if the stepsize h_k has to be changed several times in order to achieve a reduction of the value of g, the bulk of the expense is done only once, and this is the cost occurred in computing $[D\boldsymbol{f}(\boldsymbol{x}^k)]^{-1}\boldsymbol{f}(\boldsymbol{x}^k)$.

3. (monisteen tehtävä 2.3.4). Write a program implementing the ideas of trivial, secant, and tangent predictors.

   (a) Use your program to find a root of the function $\boldsymbol{f}(\boldsymbol{x}) = 0$, where

$$f_i(\boldsymbol{x}) = (i + \sum_{k=i}^{n} x_k^3)$$

   for $n = 10$. (Observe how we embedded this nonlinear equation in a parametrized problem by using the embedding of Example 3.3).

   (b) With the same embedding as in Example 3.3, find a root of $\boldsymbol{f}(\boldsymbol{x}) = 0$, where now

$$f_1(\boldsymbol{x}) = x_1 - \prod_{j=1}^{n} x_j, \quad f_i(\boldsymbol{x}) = n - i + 1 - \sum_{j=i}^{n} x_j, \quad i = 2, \ldots, n.$$

   Do it for $n = 10$ and $n = 25$. (For comparison, exact solution here is $\boldsymbol{x}(1) = (1, \ldots, 1)^T$).

4. (monisteen tehtävä 3.1.1). Use the `Matlab` program `Wu_g.m` to approximate the local stable and unstable manifolds of one of the fixed points of the Henon map

$$\phi(x,y) = \begin{bmatrix} 1+y-ax^2 \\ bx \end{bmatrix}.$$

Use parameter values $a = 1.4$, $b = 0.3$.

5. (monisteen tehtävä 3.1.5). Using Taylor expansion, obtain another approximation for the local unstable manifold of the Henon map in Problem 4. Also draw a comparing plot between the results obtained here and those obtained using the graph transform.

6. (monisteen tehtävä 3.1.7). Consider the map

$$\phi(x,y) = \begin{bmatrix} x+y+cx(x-1) \\ y+cx(x-1) \end{bmatrix},$$

where $0 < c < 4$. This has two fixed points. Do a graphical study of the stable and unstable manifolds of the hyperbolic fixed point, for the values $c = 0.5, 1, 1.5, 2$, by plotting them on the same figure. Also, observe that the determinant of the Jacobian of the map is 1 for all $(x,y) \in \mathbb{R}^2$. What does this mean?