Harjoitus 2 on tietokoneharjoitus. Tehtäviä tehdään yhdessä assistentin kanssa tietokoneluokassa ja joistain tehtävistä palautetaan lyhyt selostus seuraavan viikon harjoituksiin mennesä.

1. (Problem 5.12 p.115) Suppose you know two linearly independent vectors in $N(\boldsymbol{f}_{\boldsymbol{u}}(\boldsymbol{u}_0))$. How can you find an approximation of the quadratic form of Theorem 4.2 with 9 evaluations of $\boldsymbol{f}$? Can you do it with less?

2. (Problem 5.19 p. 116) Compute the bifurcation diagram for the following system depending on the parameter c:
$$\begin{cases} x' &= y - y^2 - x(x^2 - y^2 + 2y^3/3 + c) \\ y' &= x + (y - y^2)(x^2 - y^2 + 2y^3/3 + c) \end{cases}.$$

   Find all Hopf and Fold-bifurcations.

3. (Example 4.12 s. 110) Consider the system of Example 4.1:
$$\begin{cases} x_1' = -x_1 + \alpha_1(1 - x_1)\, e^{x_2} \\ x_2' = -x_2 + \alpha_1\alpha_2(1 - x_1)\, e^{x_2} - \alpha_3 x_2. \end{cases}$$

   For $\alpha_1 = 0$, the origin is an equilibrium. Continue from this letting α_1 be the free parameter and fixing $\alpha_2 = 14$, $\alpha_3 = 2$.

   Palauta (23.2. Mennessä) lyhyt selostus. Yritä piirtää s.111 - mukainen kuva sekä tarkastele foldeissa kummalla puolella stabiilit tasapainopisteet ovat? ja hopfeissa syntyvien periodisten ratkaisujen stabiiliutta?

4. (Problem 5.21 p.116) We want to understand the bifurcation diagram for the following biological model:
$$\begin{aligned} x_1' &= x_1(1 - x_1) - \alpha_2 x_1 x_2 \\ x_2' &= -x_2/4 + \alpha_2 x_1 x_2 - 3x_2 x_3 - \alpha_1(1 - e^{-5x_2}) \\ x_3' &= -x_3/2 + 3x_2 x_3. \end{aligned}$$

   At first, let $\alpha_2 = 3$ and let α_1 be free; restrict α_1 as $0 \le \alpha_1 \le 0.6$. Then, continue the equilibrium $(1, 0, 0)$ from $\alpha_1 = 0$, and find the bifurcation diagram for this problem. Inspect the diagram. You should observe the branch point at the initial point, and then a fold and one more branch point and two Hopf points on the new branch. Now, continue the fold and the second Hopf point in two parameters by freeing also α_2. Do these continuations on both directions. Append the results to the previous ones, and inspect them.