Teknillinen korkeakoulu
Johdatus solmuteoriaan, kevät 2002
Exercise 8. (22.3.)

1. Calculate the fundamental group of the surface below by using the Seifert–van Kampen theorem. There will be four generators and one relation in the end. Hint: The generators and relations for the surface {torus\small disc} are given in the copies given.

2. a) Find a presentation for the group of the knot 4_1 by using the Wirtinger method.
   b) Choose one of the ralations and show that it follows from the others.
   c) Find a presentation with only two generators and one relation.

3. a) Study the relations in the Wirtinger method and show that after abelianization (add relations $a_i a_j = a_j a_i$), the group of a PL-knot will become a presentation for the group $\mathbf{Z}$. In other words, $H_1(\mathbf{R}^3 \setminus S) \cong \pi(S)_{\mathrm{ab}} \cong \mathbf{Z}$. What kind of a loop will generate this group?
   b) Show that the loop in the figure below is **homologically unlinked** with the knot 4_1.
   Hint: Present the loop in terms of the generators in problem 2. Definition below.

4. For the link below, find a presentation consisting of three generators and two relations.

Definition. Let K be a link and let S be a knot disjoint from K. We say that S is *homotopically unlinked* with K if the homotopy class in $\pi_1(\mathbf{R}^3 \setminus K)$ corresponding to S is trivial in the group $\pi_1(\mathbf{R}^3 \setminus K)$. Similarly, S is *homologically unlinked* with K if the homology class corresponding to S is trivial in the group $H_1(\mathbf{R}^3 \setminus K)$.