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Exercise 2. (1.2.)

1. Show that the homeomorphism
   $f\colon \mathbf{R}^2 \to \mathbf{R}^2$, $f(r,\varphi) = (r^2,\varphi)$, defined in polar coordinates, is isotopic to the identity map.

2. Let $f\colon \mathbf{S}^1 \to \mathbf{S}^1$ be a homeomorphism such that the function $\varphi \mapsto \arg f(e^{i\varphi})$ is increasing. Show that f is isotopic to the identity map.

3. Let $f\colon \overline{\mathbf{B}}^2 \to \overline{\mathbf{B}}^2$ be a homeomorphism with $f(z) = z$ for all $z \in \mathbf{S}^1$. Show that f is isotopic to the identity map of $\overline{\mathbf{B}}^2$ by using the formula

   $$H(z,t) = \begin{cases} z, & \text{for } t \le |z| \le 1, \\ tf(z/t), & \text{for } |z| \le t, \end{cases}$$

   for values $0 < t \le 1$, and setting $H(z,0) = z$ for all $z \in \overline{\mathbf{B}}^2$. Exercise 1 from last time may be helpful.

4. Let f, $g\colon \overline{\mathbf{B}}^2 \to \overline{\mathbf{B}}^2$ be homeomorphisms with $f(z) = g(z)$ for all $z \in \mathbf{S}^1$. Show that f and g are isotopic.
   Hint: Consider first $f^{-1} \circ g$. How does it behave on the boundary?