Teknillinen korkeakoulu
Johdatus solmuteoriaan, kevät 2002
Exercise 11. (19.4.)

Note: Some (unproved) theorems from the lectures are needed to solve the problems.

All the knots are assumed to be PL.

1. a) Deduce from the knot table, how do the knots 9_1, $11_1, \ldots$ look like.
   b) Deduce that in the case of an even crossing number, the corresponding diagrams present a 2-component link.
   c) Let $P_{2n+1}(t)$ be the Jones polynomial of the knot $(2n+1)_1$. Find a recurrence relation, which makes it possible to compute these polynomials.
   Hint: Use the formula from Problem 2 of the last week as well as the Jones polynomial Q_{2n} from part b.
   d) Compute $V(5_1)$ and $V(7_1)$ and check the result from the table overleaf.

2. Using the previous problem and the results from the lectures, show that there are infinitely many prime knots, such that no two of them are mutually equivalent.

3. a) Using Tait's II Conjecture, show that $w(K) = 0$ whenever K is a reduced and alternating diagram of an amphicheiral and alternating knot.
   b) Let S be an alternating knot with an odd crossing number. Show that S cannot be amphicheiral.

4. a) Show that the sum of two alternating knots is alternating. (See the example in the lectures)
   b) Let $S_1 + S_2$ be an alternating knot. Show that S_1 and S_2 are both alternating.
   c) Let S_1 and S_2 be alternating knots. Show that $c(S_1 + S_2) = c(S_1) + c(S_2)$. Here c is the crossing number.
   d) Show that the only knot S with an inverse S^{-1}, is the trivial knot.
   Hint: The inverse should satisfy $S + S^{-1} = O$. Note that the trivial knot is alternating.
   (It is not known whether part c is true for all knots)