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Exercise 10. (12.4.)

1. Calculate the Jones polynomial for the knot 4_1 and also for the Whitehead link.

2. a) Consider three diagrams L_0, L_1 and L_2 that differ from each other in one crossing only, as in the figure below.

   Show that
$$\frac{1}{t}V(L_1) - tV(L_2) + (\frac{1}{\sqrt{t}} - \sqrt{t})V(L_0) = 0.$$
   b) Compute $V(5_2)$ using part a.
   c) Show that $V_S(1) = 1$ for all PL knots S. (Hint: Prob. 4/Exe. 3)

3. a) Consider an oriented link with two components A_+ and B_+. Show that
$$V(A_+ \cup B_-) = t^{\alpha}V(A_+ \cup B_+),$$
   where $\alpha = -3 \cdot \mathrm{lk}(A_+, B_+)$.
   b) Consider a diagram with two adjacent crossings as below. Show that, when computing the Kauffman polynomial using property (iii), the result stays the same, even if (iii) is applied in a different order to the crossings.

4. a) Let S_1 and S_2 be PL knots. Justify the equation $V(S_1+S_2) = V(S_1) \cdot V(S_2)$. Hint: Start calculating with S_1.
   b) Show that the Square knot $3_1 + \rho(3_1)$ and the Granny knot $3_1 + 3_1$ are not equivalent.