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Exercise 1. (25.1.)

1. a) Prove: If $A_1,\ A_2 \subset \mathbf{R}^n$ are closed sets and $f_1\colon A_1 \to \mathbf{R}^n$, $f_2\colon A_2 \to \mathbf{R}^n$ are continuous functions with $f_1(x) = f_2(x)$ whenever $x \in A_1 \cap A_2$, then the formula
$$f(a) = \begin{cases} f_1(a), & \text{kun } a \in A_1, \\ f_2(a), & \text{kun } a \in A_2 \end{cases}$$
defines a continuous function $f = f_1 \cup f_2\colon A_1 \cup A_2 \to \mathbf{R}^n$.
b) Give an example of a situation, where the sets are not closed and the function f will be discontinuous.

2. Classify the alphabet A, B, C, ..., Å, Ä, Ö into groups with homeomorphic elements. You may choose yourself how to draw them.

3. Let $f\colon \mathbf{S}^1 \to \mathbf{S}^1$ be a homeomorphism. Show that there is a homeomorphism $F\colon \overline{\mathbf{B}}^2 \to \overline{\mathbf{B}}^2$ such that $F(z) = f(z)$ for all $z \in \mathbf{S}^1$.

   Here $\mathbf{S}^1 = \{z \in \mathbf{C} \mid |z| = 1\}$ and $\overline{\mathbf{B}}^2 = \{z \in \mathbf{C} \mid |z| \le 1\}$.

4. Let $f\colon [0,1] \to]-1,1[$ be a continuous function with $f(0) = f(1) = 0$. Define a homeomorphism $F\colon [0,1] \times [-1,1] \to [0,1] \times [-1,1]$, which
(i) keeps all boundary points fixed,
(ii) rectifies the graph of f to the line segment $[0,1]$.
It is enough to give the definition; no proofs are needed.