Helsinki University of Technology, TKK
Mat-5.3701 Analytical Mechanics
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Exercise 7, 2006 March 21, Tuesday 14-16 Y313
`http://math.tkk.fi/teaching/analmek`

1. **Home exercise:** Calculate the moment of inertia I_3 for the two arrangements of four balls shown Figure 1 two heavy ones (radius R, mass M) and two light ones (radius r, mass m) with a homogeneous mass density.

   As a model for a dancer's pirouette compare the angular velocity for the two arrangements, with L_3 fixed and equal in the two cases.

2. Calculate the moments of inertia of a torus filled homogeneously with mass, with the main radius R and the section radius r.

3. Find the kinetic energy of a homogeneous cylinder of radius a rolling inside a cylindrical surface of radius R, see Figure 2.

4. A homogeneous circular cylinder with the length h, radius r and mass m rolls along an inclined plane in the earth's gravitational field.

   (i) Construct the full kinetic energy of the cylinder and find the moment of inertia relevant to the described motion.

   (ii) Construct the Lagrangian function and solve the equation of motion.

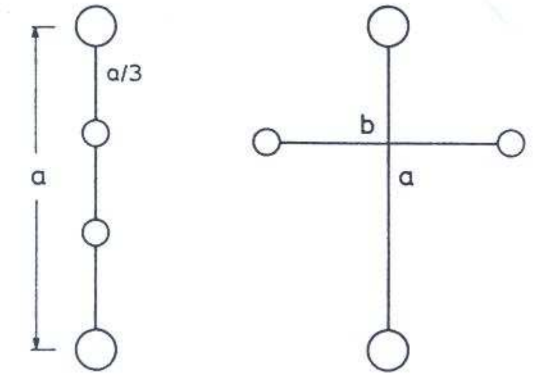

Figure 1: Two arrangements for a dancer's model

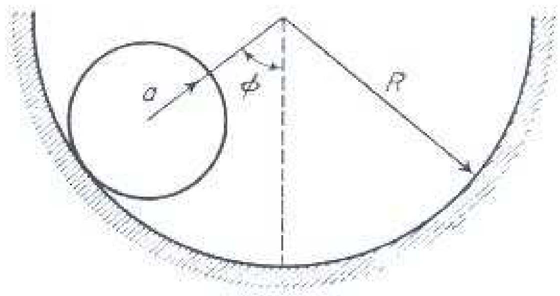

Figure 2: A rolling cylinder