Helsinki University of Technology, TKK
Mat-5.3701 Analytical Mechanics
Stenberg / Niiranen
Exercise 5, 2006 February 21, Tuesday 14-16 Y313
`http://math.tkk.fi/teaching/analmek`

1. Prove that
$$\det(\boldsymbol{I} + \epsilon \boldsymbol{A}) = 1 + \epsilon \mathrm{tr} \boldsymbol{A} + \mathcal{O}(\epsilon^2)\,.$$

2. (i) Evaluate the following Poisson brackets for a single particle:
$$\{l_i, r_k\}\,, \quad \{l_i, p_k\}\,, \quad \{l_i, \boldsymbol{r}\}\,, \quad \{l_i, \boldsymbol{p}^2\}\,.$$
   (ii) If the Hamiltonian in its "natural" form $H = T + U$ is invariant under rotations, what quantities can U depend on?

3. Making use of the Poisson brackets show that for the system $H = T + U(r)$, with $U(r) = \gamma/r$, where γ is a constant, the vector
$$\boldsymbol{a} = \boldsymbol{p} \times \boldsymbol{l} + \boldsymbol{x} m\gamma/r$$
is an integral of motion (Lenz' vector or Hermann–Bernoulli–Laplace vector).

4. **Home exercise:** A system of two degrees of freedom is described by the Hamiltonian
$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2\,.$$
Show that
$$F_1 = \frac{p_1 - a q_1}{q_2}\,, \quad F_2 = q_1 q_2$$
are constants of motion. Are there any other independent algebraic constants of motion? Can any be constructed from Jacobi's identity?