Helsinki University of Technology, TKK
Mat-5.3701 Analytical Mechanics
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Exercise 1, 2006 January 24, Tuesday 14-16 Y313
`http://math.tkk.fi/teaching/analmek`

1. **Home exercise:** (Landau, Lifshitz: Mechanics; Pr. I.5.1, I.5.2) Let's consider

   (a) a coplanar double pendulum of masses m_1 and m_2 in Fig. 1.

   (b) a pendulum of mass m_2 moving horizontally with a mass m_1 in Fig. 2.

   Write the Lagrange equations of motion. Try to solve the problem for some parameter values and initial conditions with your "favorite" software (Maple/Matlab/Mathematica/...).

2. The function $f = f(x_1, ..., x_n)$ is positive-homogeneous of degree k in $x_1, ..., x_n$ if

   $$f(\lambda x_1, ..., \lambda x_n) = \lambda^k f(x_1, ..., x_n)$$

   for all $\lambda > 0$. Prove Euler's theorem: If f is continuously differentiable and positive-homogeneous of degree k, then

   $$\sum_{i=1}^{n} \frac{\partial f}{\partial x_i} x_i = kf\,.$$

3. If in the integral

   $$\int_{x_1}^{x_2} f(y, y')dx$$

   f does not depend explicitly on x, show that

   $$y' \frac{\partial f}{\partial y'} - f(y, y')$$

   is constant. Apply this result to $L(q, \dot{q}) = T - U$ and identify the constant. T is assumed to be a homogeneous quadratic form in $\dot{q}$.

4. For $\boldsymbol{x} = (x_1, ..., x_n)^T$ let the kinetic and potential energies be

   $$T = \frac{1}{2}\dot{\boldsymbol{x}}^T \boldsymbol{M} \dot{\boldsymbol{x}}\,, \quad U = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{K} \boldsymbol{x}\,,$$

   with symmetric and positive definite $\boldsymbol{M}$ and $\boldsymbol{K}$. Write the Lagrange and Hamilton equations of motion.

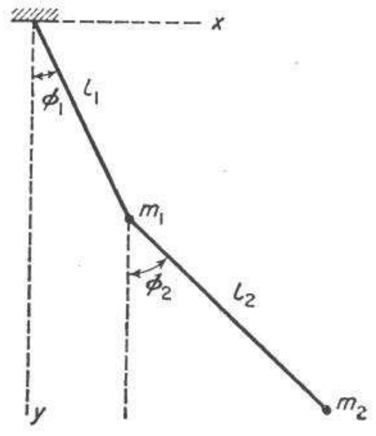

Figure 1: Coplanar double pendulum

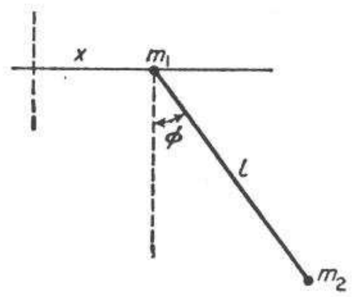

Figure 2: Moving mass pendulum