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# MATLAB II
## Exercise for Lecture 2

This exercise involves the second lecture of the minicourse Matlab continuation. The topic is **differential equations**.
Once you have solved the problems, please send **published pdf** and your **source code** to *heikki.apiola@aalto.fi*.
The "CREDIT-exercise" is the nr.4 Apollo-problem. The rest are just for extra practice.
The deadline for the return of the exercise is XX.XX.2018 (we'll discuss this during the lectures.)
I (Heikki) will be in Otaniemi also at the same time with Juha's office hours on some Fridays.

1. (a) Plot the direction field for the equation

$$y' = 3\,\sin y + y - 2$$

on a rectangle $-2 \le t \le 10, -1 \le y \le 6$.

- Looking at the dirfield, how can you see that the equation is autonomous?
- Find approximatively the constant solutions from the dirfield figure, then plot the RHS, and finally use `fzero` to find more accurate values.
- Use `ode45` to find solutions below, between and above the constant solutions. You are welcome to complete the picture using our interactive way to draw solution curves.
- Determine the stability/nonstability of the critical points (enough to look at the fieldplot).
- (Voluntary) Use `dsolve` to find the symbolic solution.

(b) The same (with "non" at suitable places) wrt, to

$$y' = y^2 - t\,y$$

$-2 \le t \le 2, -4 \le y \le 4$. (Can adjust) ** See Coombes p. 60 **

2. [Moler p. 39, problem 7.15]
A classical model in mathematical ecology is the Lotka–Volterra predator- prey model. Consider a simple ecosystem consisting of rabbits that have an infinite supply of food and foxes that prey on the rabbits for their food. This is modeled by a pair of nonlinear, first-order differential equations:

$$\begin{cases} r' = 2r - \alpha\, r\, f, r(0) = r_0 \\ f' = -f + \alpha\, r\, f, f(0) = f_0 \end{cases}$$

$r(t) =$ Nr. of rabbits, $f(t) =$ Nr. of foxes, $\alpha > 0$.
It turns out that the solutions are always periodic, with a period $t_p$ that depends on the initial conditions.

(a) Compute the solution with $r_0 = 300, f_0 = 150, \alpha = 0.01$. You should find that $t_p$ is close to 5. Make two plots:
$r(t)$ and $f(t)$ on time-axis and phase plane: axes $(r, f)$.

(b) Compute and plot the solution with $r_0 = 15, f_0 = 22, \alpha = 0.01$ You should find that $t_p$ is close to 6.62.

(c) $r_0 = 102, f_0 = 198, \alpha = 0.01$. Determine the period $t_p$ by trial and error or by event handling, or ...

(d) The point $(1/\alpha, 2/\alpha)$ is a stable equilibrium point. If the populations have these initial values, they do not change. If the initial populations are close to these values, they do not change very much ... ** linearization, forget ***

3. [7.16]
Many modifications of the Lotka–Volterra predator-prey model (see previous problem) have been proposed to more accurately reflect what happens in nature. For example, the number of rabbits can be prevented from growing indefinitely by changing the first equation as follows:

$$\begin{cases} r' = 2(1 - \frac{r}{R})r - \alpha\, r\, f, r(0) = r_0 \\ f' = -f + \alpha\, r\, f, f(0) = f_0 \end{cases}$$

...Consequently, the number of rabbits can never exceed $R$.

.. compare ...

4. Apollo: from the book: Forsythe-Malcolm-Moler: Computer methods for mathematical computations, Prentice Hall 1977 (Restricted 3-body-problem)
(Reminds me to the movie: “Hidden figures”)
The following differential equations describe the motion of a body in orbit about 2 much heavier bodies. An example: Apollo capsule in an earth-moon orbit The 3 bodies determine a plane in space. The coordinate system: x-axis is the line through moon and earth. The origin is at the center of mass of the 2 heavy bodies. If $\mu = \frac{m}{M}$ is the ratio of the masses, then the moon and the earth are located at $(1 - \mu, 0)$ and $-\mu$, as we take the distance between the bodies as unity. The Apollo is assumed to habve a negligible mass compared to the other 2. Let $(x(t), y(t))$ be the position of Apollo. The Newton laws of motion are:

$$\begin{cases} x'' = 2y' + x - \frac{\lambda(x+\mu)}{r_1^3} - \frac{\mu(x-\lambda)}{r_2^3} \\ y'' = -2x' + y - \frac{\lambda y}{r_1^3} - \frac{\mu\, y}{r_2^3} \end{cases}$$

$\mu = \frac{1}{82.45}, \lambda = 1 - \mu$
$r_1 = ((x + \mu)^2 + y^2)^{1/2}, r_2 = ((x - \lambda)^2 + y^2)^{1/2}$

The first derivatives come from the rotating coordinate system.

One type of problems is to study periodic solutions. It is known that the initial conditions $x(0) = 1.2, x'(0) = 0, y(0) = 0, y'(0) = -1.04935751$

leads to a solution which is periodic with period $T = 6.19216933$.

These conditions mean that Apollo starts at the far side of the moon at altitude about 0.2 times the earth-moon distance, The resulting orbit brins Apollo in fairly close to the earth, out in a big loop on the opposite side of the earth from the moon, back in close to the earth again, and finally back to its original position pon the far side of the moon.

(a) Compute and plot this solution using `ode45` and verify periodicity.
(b) How close does Apollo come to the earth's surface
(c) Determine the stepsizes of the T-vector, what is the minimum and where.
(d) Try different values of the error tolerances.
(e) Animate the flight.
(g) Find something else, and have fun!