“Harmonic Analysis”

“Outline of the lecture for Friday 24-April-2015”

• We will use this result to sketch the celebrated vector-valued extension of the Hardy-Littlewood maximal extension:

$$\int_{\mathbb{R}^n} \overline{M}_q f(x)^p \, dx \leq C \int_{\mathbb{R}^n} |f(x)|_q^p \, dx.$$

where

$$\overline{M}_q f(x) = \left(\sum_{i=1}^{\infty} (M f_i(x))^q \right)^{1/q}.$$

and

$$|f(x)|_q = \left(\sum_{i=1}^{\infty} |f_i(x)|^q \right)^{1/q} = \|f(x)\|_{\ell^q}$$

we have for $1 < p < \infty$ and $1 < q \leq \infty$.

• We plan to recall the definition of the A_1 class of weights and prove the Coifman-Rochberg theorem: $(M(\mu))^\delta \in A_1$ if $0 < \delta < 1$. Give some examples after that.

• We also plan to build more examples of A_1 weights by means of the so called Rubio de Francia’s algorithm, a key tool in this part of Harmonic Analysis.

• Definition of the A_p class of weights of Muckenhoupt. Relationship with the A_1 class of weights. Characterizations of these class of weights. Factorization.

• The reverse Hölder property for these class of weights and the ”openness” property.