"Harmonic Analysis"

"Outline of the lecture for Tuesday 21-April-2015"

- We plan to prove the Fefferman-Stein theorem

$$\|Mf\|_{L^{1,\infty}(w)} \leq c_n \int_{\mathbb{R}^n} |f(x)|\, Mw(x) dx.$$

We will give two proofs being one of them based on the Besicovtch lemma. As a consequence we will prove that if $1 < p < \infty$ there exists a constant C such that for all f

$$\|Mf\|_{L^p(w)} \leq c_n\, p' \, \|f\|_{L^p(Mw)}.$$

- We will use this result to sketch the celebrated vector-valued extension of the Hardy-Littlewood maximal extension:

$$\int_{\mathbb{R}^n} \overline{M}_q f(x)^p \, dx \leq C \int_{\mathbb{R}^n} |f(x)|_q^p \, dx.$$

where

$$\overline{M}_q f(x) = \left(\sum_{i=1}^{\infty} (Mf_i(x))^q \right)^{1/q}.$$

and

$$|f(x)|_q = \left(\sum_{i=1}^{\infty} |f_i(x)|^q \right)^{1/q} = \|f(x)\|_{\ell^q}$$

we have for $1 < p < \infty$ and $1 < q \leq \infty$.

- Definition of the A_1 class of weights.

- Definition of the A_p class of weights of Muckenhoupt. Relationship with the A_1 class of weights.