Institute of mathematics
Helsinki University of Technology
Arponen

Mat-1.3651 Numerical Linear Algebra (Numeerinen matriisilaskenta)

Final examination 17.04.2008

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Study programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KJO, KTA, KON, MAK, MAR, PUU, RAK, TFY, TIK, TLT, TUO, YHD.

Calculators are not allowed nor needed. Time for the exam is 3 hours. You can answer in Finnish if you wish.

This exam constitutes 75% of the final grade.
The home assignments constitute 25%.

1. Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 6 & 9 \\ 2 & 8 & 9 \end{pmatrix}$. Use the Gaussian elimination with partial pivoting to compute P, L, and U s.t. $PA = LU$ where L is unit lower triangular, U upper triangular and P a permutation. (Hint: U should be integers now!)

2. Let $A \in \mathbb{R}^{m\times m}$ be tridiagonal and symmetric.
   (a) In the QR factorization $A = QR$, which entries of R are nonzero?
   (b) Show that the tridiagonal structure is recovered when the product RQ is formed.

3. Let $A \in \mathbb{C}^{m\times m}$. Show that
   (a) there exist (column) vectors $u_j, v_j \in \mathbb{C}^m$, $j = 1, \dots, m$ such that
   $$I - zA = (I - zu_m v_m^*) \cdots (I - zu_2 v_1^*)(I - zu_1 v_1^*) \quad \forall z \in \mathbb{C}.$$
   (b) The inner products $u_j^* v_j$ are the eigenvalues of A.

   Hint: Schur.

4. Consider the problem $Ax = b$.
   (a) Explain the Arnoldi iteration
   (b) Describe how is Arnoldi related to the GMRES least squares problem
   $$\|\tilde{H}_n y - \|b\| e_1\| = \min . \tag{A}$$
   (c) Describe an $O(n^2)$ algorithm to solve (A) based on QR factorization by Givens rotations.