Summability of joint cumulants of nonindependent lattice fields

Jani Lukkarinen

joint work with

Matteo Marcozzi (U Helsinki) and Alessia Nota (U Bonn)

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UNIVERSITY OF HELSINKI

CENTRES OF EXCELLENCE IN RESEARCH

Outline

- Why cumulants instead of moments?
- "Random lattice fields"
- ℓ_p-clustering norms of cumulants
- Main theorem:
  Summability of joint correlations of two random fields
- An Example and a Counterexample
- Example 2: Discrete nonlinear Schrödinger equation
- Main tools: Wick polynomials and cluster expansions
- Outline of the proof
- Weighted ℓ_2-clustering result

Some shorthand notations

- Consider *random variables* y_j, $j = 1, 2, \ldots, N$ on a *probability space* $(\Omega, \mathcal{B}, \mu)$
- For any sequence $J = (j_1, \ldots, j_n)$ of indices, denote the *product variable* by

$$y^J = y_{j_1} y_{j_2} \cdots y_{j_n} = \prod_{k=1}^{n} y_{j_k}$$

- The corresponding *moment* is

$$\mathbb{E}[y^J] = \mathbb{E}[y_{j_1} y_{j_2} \cdots y_{j_n}]$$

- If joint exponential moments exist ($\langle e^{\beta \sum_j |y_j|} \rangle < \infty$, $\beta > 0$), can differentiate a *moment generating function*,

$$g_{mom}(\lambda) := \mathbb{E}[e^{\lambda \cdot y}] \quad \Rightarrow \quad \mathbb{E}[y^J] = \partial_\lambda^J g_{mom}(0)$$

What are cumulants?

- Cumulants $\kappa[y_J]$ may be defined *recursively* from the identity (choose any $j \in J$)

$$\mathbb{E}[y^J] = \kappa[y_J] + \sum_{E : j \in E \subsetneq J} \mathbb{E}[y^{J\setminus E}]\, \kappa[y_E]$$

- If exponential moments exist, obtained also from a *cumulant generating function*: with $\partial_\lambda^J := \prod_{j\in J} \partial_{\lambda_j}$,

$$g(\lambda) := \ln g_{mom}(\lambda) \quad \Rightarrow \quad \kappa[y_J] = \partial_\lambda^J g(0)$$

- Cumulants are *multilinear* and *permutation invariant*
- *Centering* only affects the first order cumulant: ($\bar{y}_j = \mathbb{E}[y_j]$)

$$g(\lambda) = \lambda \cdot \bar{y} + \ln \mathbb{E}[e^{\lambda\cdot(y-\bar{y})}]$$

Random lattice fields

- Let Z be a *countable* index set, e.g., a lattice $Z = \mathbb{Z}^d$
- We consider here *complex lattice fields* $\psi(x)$, $x \in Z$, i.e., a *countable collection of random variables*
- For simplicity, we assume that the field is *closed under complex conjugation*:

  To every $x \in Z$ there is some $x_* \in Z$ for which $\psi(x)^* = \psi(x_*)$

  (It is always possible to satisfy this by augmenting the index set to $Z \times \{1, -1\}$, $\psi(x, 1) = \psi(x)$, $\psi(x, -1) = \psi(x)^*$)
- In addition to concrete examples from physics, covers *also abstract index sets*, such as the sequence of coefficients in the Karhunen–Loève decomposition of a stochastic process

Why study cumulants of random fields?

Observation: If y, z are independent random variables we have

$$\mathbb{E}[y^n z^m] = \mathbb{E}[y^n]\mathbb{E}[z^m] \neq 0$$

whereas the corresponding cumulant is zero if $n, m \neq 0$.

Consider a *random lattice field* $\psi(x)$, $x \in \mathbb{Z}^d$, which is (very) strongly mixing under lattice translations:

Assume the fields in well separated regions become asymptotically independent as the separation grows.

- Then $\kappa[\psi(x), \psi(x+y_1), \ldots, \psi(x+y_{n-1})] \to 0$ as $|y_i| \to \infty$. How fast? ℓ_1- or ℓ_2-summably?
- Not true for corresponding moments: $\mathbb{E}[|\psi(x)|^2|\psi(x+y)|^2]$

ℓ_p-clustering fields

ℓ_p-clustering norm of a random field ψ

Suppose ψ is a random field on a countable Z. Define

$$\|\psi\|_p^{(n)} := \sup_{x_0 \in Z} \left[\sum_{x \in Z^{n-1}} |\kappa[\psi(x_0), \psi(x_1), \ldots, \psi(x_{n-1})]|^p \right]^{1/p}$$

We call the field ℓ_p-clustering if $\|\psi\|_p^{(n)} < \infty$ for all n

- Involves the *n:th connected correlation function*

$$u_n(x_1, \ldots, x_n) = \kappa[\psi(x_1), \ldots, \psi(x_n)]$$

- We can measure the *magnitude* of the field with

$$M_N(\psi; p) := \max_{1 \le n \le N} \left(\frac{1}{n!} \|\psi\|_p^{(n)} \right)^{1/n}$$

- If $1 \le p \le 2$ and ψ is ℓ_p-clustering on $Z = \mathbb{Z}^d$, can take Fourier-transform in the displacement $y_i = x_i - x_0$

  $\Rightarrow$ function $F^{(n)}(x_0, k)$ is L^∞ in $x_0 \in \mathbb{Z}^d$ and L^2-integrable in $k \in (\mathbb{T}^d)^{n-1}$

- ℓ_1-clustering implies that $F^{(n)}(x_0, k)$ is continuous and uniformly bounded ($\Rightarrow$ helps in nonlinearities)

- If the field has a *translation invariant* distribution on $Z = \mathbb{Z}^d$, a change of variables $y_i = x_i - x_0$ yields

$$\|\psi\|_p^{(n)} = \left[\sum_{y \in (\mathbb{Z}^d)^{n-1}} |\kappa[\psi(0), \psi(y_1), \ldots, \psi(y_{n-1})]|^p \right]^{1/p}$$

- Examples of *ℓ_1-clustering* thermal Gibbs states:
  discrete NLS [Abdesselam, Procacci, Scoppola],
  certain fermionic lattice systems [Salmhofer],
  any state which has an *exponential decay of correlations*, ...

Main result

Suppose that

1. Z is a countable index set and $N \in \mathbb{N}_+$
2. $\phi(x)$ and $\psi(x)$ two random fields on Z, closed under conjugation and defined on the same probability space
3. ϕ is ℓ_1-clustering up to order $2N$
4. ψ is ℓ_∞-clustering up to order $2N$

ℓ_2-summability of joint cumulants

For any $n, m \in \mathbb{N}_+$ for which $n, m \leq N$,

$$\sup_{x' \in Z^m} \left[\sum_{x \in Z^n} \left| \kappa[\psi(x'_1), \ldots, \psi(x'_m), \phi(x_1), \ldots, \phi(x_n)] \right|^2 \right]^{1/2} \leq (\mathfrak{M}_{m,n} \gamma^m)^{n+m} (n+m)!$$

where $\mathfrak{M}_{m,n} := \max(M_{2m}(\psi; \infty), M_{2n}(\phi; 1))$ and $\gamma = 2e \approx 5.44$.

Example 1: translation invariant Gaussian lattice fields

Define random fields ψ and ϕ on $Z = \mathbb{Z}$ such that

1. Both fields have zero mean: $\mathbb{E}[\psi(x)] = 0 = \mathbb{E}[\phi(x)]$
2. They form a collection of jointly Gaussian random variables with
$$\mathbb{E}[\psi(x)\psi(y)] = F_1(x-y)\,, \quad \mathbb{E}[\phi(x)\phi(y)] = F_2(x-y)$$
$$\mathbb{E}[\psi(x)\phi(y)] = G(x-y)\,, \qquad (x, y \in \mathbb{Z})$$
3. The covariance functions $F_1,\ F_2,\ G \in \ell_2(\mathbb{Z}, \mathbb{R})$
$\Rightarrow$ there are Fourier transforms $\widehat{F}_1,\ \widehat{F}_2,\ \widehat{G} \in L^2(\mathbb{T})$

Then the covariance operator is positive semi-definite if

$$\widehat{F}_1(k) \geq 0\,, \quad \widehat{F}_2(k) \geq 0\,, \quad \left|\widehat{G}(k)\right|^2 \leq \widehat{F}_1(k)\widehat{F}_2(k)$$

$\Rightarrow$ a unique translation invariant Gaussian measure on functions on $\mathbb{Z}$

- Here: ψ and ϕ are ℓ_2-clustering $\Rightarrow$ joint correlations ℓ_2-summable
- If ψ and ϕ are ℓ_1-clustering, are their correlations ℓ_1-summable?

Counterexample for "ℓ_1-clustering $\Rightarrow$ ℓ_1-summability"

In the above, choose ψ and ϕ *i.i.d. Gaussian*:

$$F_1(x) = \mathbb{1}_{\{x=0\}} = F_2(x)\,, \quad G(x) = \frac{1}{\pi x}\sin\left(\frac{\pi}{2}x\right)$$

$$\Rightarrow \quad \widehat{F}_1(k) = 1 = \widehat{F}_2(k)\,, \quad \widehat{G}(k) = \mathbb{1}_{\{|k|<\frac{1}{4}\}} \le 1 = \sqrt{\widehat{F}_1(k)\widehat{F}_2(k)}$$

- $\sup_{x\in\mathbb{Z}} \sum_{y\in\mathbb{Z}} |F_1(x-y)| = \|\psi\|_1^{(2)} = \|\phi\|_1^{(2)} = 1$

  & all cumulants of order $n \neq 2$ are zero

  $\Rightarrow$ Fields ψ *and* ϕ *are* ℓ_1*-clustering*
- Their joint correlations are not ℓ_1-summable!

$$\sum_{x\in\mathbb{Z}} |\kappa[\psi(x'),\phi(x)]| = \sum_{y\in\mathbb{Z}} |G(y)| = \frac{1}{2} + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1} = \infty$$

- In contrast,

$$G \in \ell^2(\mathbb{Z}) \quad \Rightarrow \quad \sup_{x'}\sum_{x} |\kappa[\psi(x'),\phi(x)]|^2 = \sum_{y}|G(y)|^2 < \infty$$

Example 2: Discrete nonlinear Schrödinger equation

The discrete NLS equation on the lattice $\mathbb{Z}^d$ deals with functions $\psi : \mathbb{R} \times \mathbb{Z}^d \to \mathbb{C}$ which satisfy

$$\mathrm{i}\partial_t \psi_t(x) = \sum_{y \in \mathbb{Z}^d} \alpha(x-y)\psi_t(y) + \lambda |\psi_t(x)|^2 \psi_t(x)$$

- Define $\psi_t(x,+1) = \psi(x)$, $\psi_t(x,-1) = \psi(x)^* \quad \Rightarrow$

  For each $t \in \mathbb{R}$ obtain a *random field* ψ_t *on* $Z = \mathbb{Z}^d \times \{-1, 1\}$
- Suppose ψ_0 has a distribution which is time-stationary and ℓ_1-clustering (e.g. an equilibrium Gibbs measure)

  $\Rightarrow$ Distribution of ψ_t is also ℓ_1-clustering

A priori estimates

- The previous results imply that any *time-correlation function* of the form

$$\kappa[\psi_0(0,\sigma'_1),\dots,\psi_0(0,\sigma'_m),\psi_t(x_1,\sigma_1),\dots,\psi_t(x_n,\sigma_n)]$$

  is ℓ_2-summable uniformly in time

- For instance, then for every t

$$f_t(x)=\kappa[\psi_0(0,-1),\psi_t(x,1)] \ \in \ell_2(\mathbb{Z})$$

  and $\|f_t\|_{\ell_2}\le C<\infty$ uniformly in time t

- The bound C depends only on the (stationary) initial data

How to use a priori estimates?

The evolution equations for ψ_t imply that

$$\mathrm{i}\partial_t f_t(x) = \sum_{y\in\mathbb{Z}^d} \alpha(x-y) f_t(y) + \lambda g_t(x)$$

$$g_t(x) = \mathbb{E}[{:}\psi_0(0,-1){:}\, \psi_t(x,-1)\psi_t(x,1)\psi_t(x,1)]$$

- Expand g_t in terms of cumulants of ψ_0 and ψ_t
  $\Rightarrow$ The main theorem shows that $\|g_t\|_{\ell_2} \le C$ for all t
- Since $f_t \in \ell_2(\mathbb{Z})$ can take a Fourier transform
- In Duhamel form the evolution equation reads for $t \ge 0$

$$\widehat{f_t}(k) = \mathrm{e}^{-\mathrm{i}t\widehat{\alpha}(k)}\widehat{f_0}(k) - \mathrm{i}\lambda \int_0^t \mathrm{d}s\, \mathrm{e}^{-\mathrm{i}(t-s)\widehat{\alpha}(k)}\widehat{g}_s(k)$$

$$\Rightarrow \quad \left\|\widehat{f_t} - \mathrm{e}^{-\mathrm{i}t\widehat{\alpha}}\widehat{f_0}\right\|_{L^2(\mathbb{T}^d)} \le Ct\lambda$$

- The harmonic evolution dominates the behaviour of f_t up to $t \propto \lambda^{-1}$

Standard definition of Wick polynomials

Wick polynomials (WP) of random variables y_j are defined recursively in $p_1 + p_2 + \cdots + p_n$ as:

1. If $p_1 = p_2 = \cdots = p_n = 0$, set $:y_1^{p_1} y_2^{p_2} \cdots y_n^{p_n}: = 1$
2. Otherwise, $\langle :y_1^{p_1} y_2^{p_2} \cdots y_n^{p_n}: \rangle = 0$
3. $\partial_{y_j} :y_1^{p_1} \cdots y_j^{p_j} \cdots y_n^{p_n}: = p_j :y_1^{p_1} \cdots y_j^{p_j - 1} \cdots y_n^{p_n}: \quad \forall j$

- The conditions have a unique polynomial solution
- If exponential moments exist, WP have a generating function

$$G_{\mathrm{w}}(y,\lambda) = \frac{\exp\left(\sum_{i=1}^{n} \lambda_i y_i\right)}{\langle \exp\left(\sum_{i=1}^{n} \lambda_i y_i\right) \rangle} = \mathrm{e}^{\lambda \cdot y - g(\lambda)}$$

$$\Rightarrow \quad :y_1^{p_1} y_2^{p_2} \cdots y_n^{p_n}: = \partial_{\lambda_1}^{p_1} \cdots \partial_{\lambda_n}^{p_n} G_{\mathrm{w}}(y,\lambda) \Big|_{\lambda=0}$$

Gaussian Wick polynomials

- WP have been mainly used for Gaussian fields.
  They were introduced in quantum field theory where the unperturbed measure concerns Gaussian (free) fields
- Gaussian case has significant simplifications:
  If $C_{j'j} = \kappa[y_{j'}, y_j]$ denotes the *covariance matrix*,

$$G_{\mathrm{w}}(y, \lambda) = \exp[\lambda \cdot (y - \langle y \rangle) - \lambda \cdot C\lambda/2]$$

$\Rightarrow$ Wick polynomials are *Hermite polynomials*

- The resulting orthogonality properties are used in the Wiener chaos expansion and Malliavin calculus

Basic properties of WP

- *Products to WP*

$$y^I = \sum_{U \subset I} {:}y^U{:}\, \mathbb{E}[y^{I \setminus U}] = \sum_{U \subset I} {:}y^U{:} \sum_{\pi \in \mathcal{P}(I \setminus U)} \prod_{A \in \pi} \kappa[y_A]$$

- *WP to products*

$${:}y^I{:} = \sum_{U \subset I} y^U \sum_{\pi \in \mathcal{P}(I \setminus U)} (-1)^{|\pi|} \prod_{A \in \pi} \kappa[y_A]$$

- WP are *permutation invariant* and *multilinear*
- Lowest order WP

$${:}y{:} = y - \mathbb{E}[y] = y - \kappa(y)$$

$${:}y_1 y_2{:} = y_1 y_2 - \kappa(y_1, y_2) - \kappa(y_1) y_2 - \kappa(y_2) y_1 + \kappa(y_1)\kappa(y_2)$$

Cluster expansions

For any index set J, as long as all moments $y^A = \prod_{i\in A} y_i, A \subset J$, belong to $L^1(\mu)$, the moments-to-cumulants formula holds:

$$\mathbb{E}[y^J] = \sum_{\pi\in\mathcal{P}(J)} \prod_{A\in\pi} \kappa[y_A]$$

- $\mathcal{P}(J)$ denotes the collection of *partitions* of J.
- For a partition $\pi \in \mathcal{P}(J)$, call the subsets $A \in \pi$ *clusters* or *blocks*
- Follows from the recursive definition of cumulants

Combinatorial properties of WP

Assume J is an index set such that $y^A = \prod_{i\in A} y_i$ belong to $L^1(\mu)$ for any $A \subset J$

Define polynomials $\mathcal{W}[y^J]$ inductively in $|J|$:

$\mathcal{W}[y^\emptyset] = 1$ and for $J \neq \emptyset$ require

$$\mathcal{W}[y^J] = y^J - \sum_{E \subsetneq J} \mathbb{E}[y^{J\setminus E}]\,\mathcal{W}[y^E]$$

- The solution is unique and $\mathcal{W}[y^J] = {:}y^J{:}$

Truncated moments-to-cumulants formula

$$\mathbb{E}\left[y^{J'}\,{:}y^{J}{:}\right] = \sum_{\pi\in\mathcal{P}(J'\cup J)} \prod_{A\in\pi} \left(\kappa[y_A]\mathbb{1}_{\{A\not\subset J\}}\right) \tag{1}$$

- ${:}y^J{:}$ are μ-a.s. unique polynomials of order $|J|$ such that (1) holds for every J'

Multi-truncated moments-to-cumulants formula

Suppose $L \geq 1$ is given and consider a collection of $L+1$ index sequences J', J_ℓ, $\ell = 1, \ldots, L$. Then with $I = J' \cup (\cup_{\ell=1}^{L} J_\ell)$

$$\mathbb{E}\left[y^{J'} \prod_{\ell=1}^{L} {:}y^{J_\ell}{:}\right] = \sum_{\pi\in\mathcal{P}(I)} \prod_{A\in\pi} \left(\kappa[y_A]\mathbb{1}_{\{A\not\subset J_\ell, \forall \ell\}}\right) .$$

Sketch of the proof of the main theorem

The difficult part is to show the main result with a single ψ-field. The general case follows by induction and combinatorial estimates.

We first prove that assuming

1. Z is a countable index set and $N \in \mathbb{N}_+$
2. ϕ is a random field on Z, closed under conjugation
3. ϕ is ℓ_1-clustering up to order $2N$
4. X is random variable with *finite variance*

ℓ_2-summability of joint cumulants

For any $n \in \mathbb{N}_+$ for which $n \leq N$,

$$\left[\sum_{x \in Z^n} \left|\kappa[X, \phi(x_1), \ldots, \phi(x_n)]\right|^2\right]^{1/2} \leq \sqrt{\mathrm{Cov}(X^*, X)}\, M_{2n}(\phi; 1)^n \mathrm{e}^n \sqrt{(2n)!}$$

Keys ideas

- Key idea: Represent cumulants using WP,

$$G(x) := \kappa[X, \phi(x_1), \ldots, \phi(x_n)] = \mathbb{E}[{:}X{:}\,{:}\phi(x_1)\cdots\phi(x_n){:}]$$

- Introduce test functions $f : Z^n \to \mathbb{C}$ with finite support,

$$\|f\|^2_{\phi,n} = \mathbb{E}\left[\Big|\sum_{x\in Z^n} f(x)\,{:}\phi(x)^{J_n}{:}\Big|^2\right] = \sum_{x',x\in Z^n} f(x')^* f(x)\Phi_n(x',x)$$

$$\Phi_n(x',x) = \mathbb{E}\left[{:}\phi^*(x')^{J'_n}{:}\ {:}\phi(x)^{J_n}{:}\right]\ , \quad J_n = \{1,2,\ldots,n\} = J'_n$$

- Φ_n are finite because all cumulants up to order $2n$ are finite

Consider

$$\Lambda[f] = \sum_{x\in Z^n} G(x) f(x) = \mathbb{E}\left[:X: \sum_{x\in Z^n} :\phi(x)^{J_n}: f(x) \right]$$

Then by Schwarz inequality

$$|\Lambda[f]|^2 \le \mathbb{E}\left[|:X:|^2 \right] \mathbb{E}\left[\left| \sum_{x\in Z^n} f(x) :\phi(x)^{J_n}: \right|^2 \right] = \mathbb{E}\left[|:X:|^2 \right] \|f\|^2_{\phi,n}$$

- $$\|f\|^2_{\phi,n} \le \sum_{x\in Z^n} |f(x)|^2 \times \sup_{x'\in Z^n} \sum_{x\in Z^n} |\Phi_n(x',x)|$$
- Lemma: ℓ_1-clustering of ϕ implies that $\sum_{x\in Z^n} |\Phi_n(x',x)| \le c_n$

$$\Rightarrow \quad |\Lambda[f]| \le \sqrt{c_n \mathbb{E}[|:X:|^2]}\, \|f\|_{\ell_2} = \sqrt{c_n \mathrm{Cov}(X^*, X)}\, \|f\|_{\ell_2}$$

- Thus by Riesz representation theorem $G \in \ell_2(Z^n)$ with

$$\sqrt{\sum_{x\in Z^n} |G(x)|^2} \le \sqrt{c_n \mathrm{Cov}(X^*, X)}$$

Combinatorial lemma 24

Let ϕ be a ℓ_p-clustering random lattice field up to order $2n$, $p \in [1,\infty]$ and $n \geq 1$. If ϕ is closed under conjugation, $\forall x' \in Z^n$

$$\|\Phi_n(x',\cdot)\|_{\ell_p} \leq \sum_{\pi \in \mathcal{P}(J_{2n})} \prod_{S\in\pi} \|\phi\|_p^{(|S|)} \leq M_{2n}(\phi;p)^{2n} \mathrm{e}^{2n} (2n)!$$

- This implies that $c_n = M_{2n}(\phi;1)^{2n} \mathrm{e}^{2n}(2n)!$
- The main ingredient to prove the lemma is the truncated moment to cumulants formula

$$\Phi_n(x',x) = \sum_{\pi \in \mathcal{P}(J'_n+J_n)} \prod_{S\in\pi} \Big(\kappa[\phi^*(x')_{A'}, \phi(x)_A] \times \mathbb{1}_{\{A'\neq\emptyset, A\neq\emptyset\}}\Big)_{A'=S|J'_n,\, A=S|J_n}$$

Second Theorem

Suppose that

1. Z is a countable index set and $N \in \mathbb{N}_+$
2. $\phi(x)$ and $\psi(x)$ two random fields on Z, closed under conjugation and defined on the same probability space
3. ϕ is ℓ_2-clustering up to order $2N$
4. ψ is ℓ_∞-clustering up to order $2N$

ℓ_2-summability of joint cumulants

For any $n, m \in \mathbb{N}_+$ for which $n, m \le N$, and all $x' \in Z^m, y \in Z^n$

$$\left[\sum_{x \in Z^n} |\Phi_n(y,x)| \left|\kappa[\psi(x'_1), \dots, \psi(x'_m), \phi(x_1), \dots, \phi(x_n)]\right|^2\right]^{1/2}$$

$$\le (\mathfrak{M}_{m,n}\gamma^m)^{2(n+m)}\left((n+m)!\right)^2$$

where $\mathfrak{M}_{m,n} := \max(M_{2m}(\psi;\infty), M_{2n}(\phi;2))$, $\gamma = 2e$, and

$$\Phi_n(y,x) := \mathbb{E}[:\phi(y_1)^*\phi(y_2)^* \cdots \phi(y_n)^*: :\phi(x_1)\phi(x_2)\cdots\phi(x_n):]$$

Proof

- For any finite $F \subset Z^n$, fixed $y \in Z^n$, pick as a test function

$$f(x) = \mathbb{1}_{\{x \in F\}} |\Phi_n(y,x)| G(x)^* \quad \Rightarrow$$

$$\Lambda[f] = \sum_{x \in F} |G(x)|^2 |\Phi_n(y,x)| \le \sqrt{\mathrm{Cov}(X^*, X)} \|f\|_{\phi,n} < \infty$$

- Thanks to the combinatorial lemma

$$\|f\|_{\phi,n}^2 \le \sum_{x \in F} |G(x)|^2 |\Phi_n(y,x)| \left(\sup_{x' \in Z^n} \sqrt{\sum_{x \in F} |\Phi_n(x',x)|^2} \right)^2$$

$$\le (c_n')^2 \sum_{x \in F} |G(x)|^2 |\Phi_n(y,x)| = (c_n')^2 \Lambda[f]$$

- Therefore, $\sqrt{\Lambda[f]} \le c_n' \sqrt{\mathrm{Cov}(X^*, X)}$ for any finite F

Summary

- Two ℓ_1-clustering fields have ℓ_2-summable joint correlations
- Two ℓ_2-clustering fields have weighted ℓ_2-summable joint correlations
- Works typically for equilibrium measures
  $\Rightarrow$ a priori bounds for time-correlations
- Possible applications: control of Green–Kubo formula for thermal conductivity, kinetic theory and transport equations
- Implications for point processes and continuum random fields?
- How optimal are the combinatorial constants?

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