Optimization systems to support planning processes in traffic and transportation

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DS&OR Lab
University of Paderborn
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University of Paderborn

University of the Information Society

~20,000 students, ~250 professors

Five Schools (Faculties):

I Faculty of Arts and Humanities
Department of English and American Studies, Department of Educational Science, Department of Protestant Theology, Department of German Studies and Comparative Literary Studies, History Department, Department of Social and Human Sciences, Department of Catholic Theology, Department of Art, Music, Textiles, Department of Media Studies, Department of Romance Languages

II Faculty of Business Administration and Economics
Department 1: Management, Department 2: Taxation, Accounting and Finance
Department 3: Business Information Systems
Department 4: Economics, Department 5: Business and Human Resource Education, Department 6: Law

III Faculty of Science
Department of Physics, Department of Chemistry, Department of Sports and Health

IV Faculty of Mechanical Engineering
Sixteen professorships, four interdisciplinary research facilities

V Faculty of Computer Science, Electrical Engineering and Mathematics
Department of Electrical Engineering and Information Technology, Department of Computer Science, Department of Mathematics
DS&OR Lab Paderborn

- Decision Support and Operations Research Lab University of Paderborn (since 1995)
  - Optimization/simulation models and applications for traffic, transportation, logistics, production, supply chain management, infrastructure networks
  - Embedded in Decision Support Systems

- PACE – International Graduate School
  - Research projects with PhD candidates
  - Mathematical optimization in production and logistics processes
  - Joint projects with enterprises
Operations Research in Germany

- German OR Society: 1300 Members
  - President 2015-16 Leena Suhl
  - 15 working groups
  - International annual conference (in English)

- Many OR professors have a chair for
  - Optimization in mathematics
  - Production management
  - Business information systems
  - Analytics
  - Controlling
Agenda

• Optimization systems; Decision Support Systems
• Application areas
• Planning problems in public transport
• Integrated vehicle and crew scheduling
• Maintaining regularity
• Integrated crew scheduling and rostering
Typical Research Topics

- Business process analysis
- Modeling approach
- Solution methods
  - Optimization, (meta)heuristics, simulation
- Special aspects such as
  - Uncertainties
  - Missing data
  - Robustness
  - Dynamics => online optimization
  - Integration
  - Multiple criteria
Decision Support System

Real problem

Modeling (Abstraction)

Model generation

Solution method

Solution / Decision proposal

Operative Data

Method Library

Visualization components

"Operations Research inside"

Further iterations if needed

Solution of the real problem

Interpretation and Implementation

Application Logic and Parameter
Optimization System

A Decision Support System able to generate and process optimization models and solutions that solve complex decision problems according to given objective(s).
Some Optimization Applications

Focus: Efficient resource utilization

- Vehicle routing and scheduling
- Production planning
- Production network optimization
- Inbound logistics optimization
- Crew scheduling
- Supply chain management
- Packing problems
- Home health care
- Water/Gas networks
- Mobile robot fulfillment systems
Planning Process in Public Transit

1. **lines + frequencies**
   - Timetabling
     - Timetable
   - Service trips
     - Vehicle blocks
       - Tasks
         - Relief points
   - Crew scheduling
     - Crew duties
   - Crew rostering
     - Crew rosters
2. **Work regulations**
3. **Crew (group) info.** (qualification, account, historical rosters, preferences)
4. **Other activities** (planned reserves, training)

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- Timetable/service trips
- Vehicle blocks/tasks
- Crew duties
- Crew rosters
Decision Support for Public Transit: Some research problems

- Multi-depot VSP, several vehicle types
- Regularity of schedules
- Integrated vehicle and crew scheduling
- Integrated crew scheduling & rostering
- Cyclic crew scheduling
- Limited #line changes
- Maintenance routing
- Robust planning
- Stochasticity
- Decision support tools
Decision Support for Public Transit: Some research problems

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Vehicle scheduling for public transport

Simple VSP:
- Construct a collection of vehicle runs for a given timetable, so that trips can be linked only through vehicle connections at terminal stations
  - Minimize the number of vehicles needed
  - Min-cost network flow problem, easily solvable

Extensions:
- Deadheading
- Multiple depots
- Periodicity
- Multiple vehicle types
- Time windows
- Maintenance routing
The Multi-Depot Vehicle Scheduling Problem (MDVSP)

Set of trips → vehicle blocks

Vehicle block:

depot A B B C A D E B depot

Deadheads (empty trips)
Crew Scheduling (after Vehicle Scheduling)

Relief point: location where a change of driver can occur
Task: portion of work between two consecutive relief points along a bus block
Crew Scheduling (after Vehicle Scheduling)

Consider: Piece of work related and duty related constraints
- Number of pieces, Min and max piece duration, min and max break duration, Min and max duty length, Min and max working time
Integrated Vehicle and Crew Scheduling

- lines + frequencies
  - timetabling
    - timetable
  - service trips
  - vehicle scheduling
    - vehicle blocks
    - tasks
    - relief points
  - crew scheduling
    - crew duties
  - crew rostering
    - crew rosters

- timetable/service trips
- vehicle blocks/tasks
- crew duties
- crew rosters
Integrated Vehicle and Crew Scheduling

- Disadvantages of sequential planning
  - Deadheads are fixed through the VSP
  - CSP may be unfeasible or not efficient

- Advantages of integration
  - Parallel consideration of VSP and CSP
  - All possible deadheads are available
  - More degrees of freedom for the CSP

- But: Problem with integration
  - Fully integrated models are large and very difficult to solve
Integrated Multi-Depot Vehicle and Crew Scheduling Problem (MDVCSP)

- **Given:** set of service trips of a timetable and set of relief points
- **Task:** find a set of vehicle blocks and crew duties such that
  - Vehicle and crew schedules are feasible
  - Vehicle and crew schedules are mutually compatible
  - Sum of vehicle and crew costs is minimized

- **Exact Formulation:** MDVSP + CSP + linking constraints
  - Compare with variable fixing heuristic
Basic Model Types

Models for the MDVSP
• Connection based flow modeling
• Time-space network flow modeling
  – Single commodity vs. Multi-commodity flow
• Set partitioning models

Models for the CSP
• Set partitioning models
• Time-space network flow modeling
  – Only for smaller problems (because of history-based restrictions)
MDVSP: Connection Based Modeling (traditional)

- Nodes ↔ Trips (n trips)
- Arc (i,j): Connection between trips i and j

# arcs: $O(n^2)$
MDVSP: Time-Space Network Modeling

- Nodes $\Leftrightarrow$ Points in time-space; Arcs $\Leftrightarrow$ trips or waiting
- $\#\text{arcs}: O(nm)$
  - $n$ trips; $m$ stations: Note that $m<<n$ !!
- Works well for the MDVSP
- Size can be drastically reduced through aggregation of arcs
Crew Scheduling: Set Partitioning Model

- Complex working time rules
  => need to follow the path of each crew member

Set partitioning

- 1) Generate a large amount of feasible duties
  - For example with resource constrained shortest path (RCSP) formulation

- 2) Use integer programming formulation:
  - Possible duties are expressed as columns of the coefficient matrix indicating which trips are covered by the duty
  - 0/1 Variable $x_j$ indicates if crew schedule $j$ is chosen or not
  - Constraints require that each trip is covered

\[
\begin{align*}
\text{Set Partitioning Problem} \\
\max & \quad \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \\
\text{s.t.} & \quad \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1, \\
& \quad \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 = 1, \\
& \quad \delta_2 + \delta_4 + \delta_5 + \delta_7 = 1, \\
& \quad \delta_4 + \delta_5 + \delta_8 = 1, \\
& \quad \delta_1 + \delta_9 = 1, \\
& \quad \delta_2 + \delta_{10} = 1, \\
& \quad \delta_3 + \delta_{11} = 1, \\
& \quad \delta_i \in \{0, 1\}, 1 \text{ to } 11
\end{align*}
\]
MDVCSP: Connection-based Formulation

Edge connecting task \( i \) and \( j \) with vehicle from depot \( d \)

\[
\min \sum_{d \in D} \sum_{(i,j) \in A^d} c_{ij}^d y_{ij}^d + \sum_{d \in D} \sum_{k \in K^d} f_k^d x_k^d
\]

\[
\sum_{d \in D} \sum_{\{i,j\} \in A^d} y_{ij}^d = 1 \quad \forall i \in N
\]

\[
\sum_{d \in D} \sum_{\{i,j\} \in A^d} y_{ij}^d = 1 \quad \forall j \in N
\]

Vehicle scheduling

Crew scheduling

\[
\sum_{\{i,j\} \in A^d} y_{ij}^d = \sum_{\{i,j\} \in A^d} y_{ji}^d \quad \forall d \in D, \forall i \in N^d
\]

\[
\sum_{\{i,j\} \in A^d} y_{ij}^d = \sum_{k \in K^d(i)} x_k^d \quad \forall d \in D, \forall i \in N^d
\]

\[
y_{ij}^d = \sum_{k \in K^d(i,j)} x_k^d \quad \forall d \in D, \forall (i,j) \in A^{sd}
\]

\[
y_{ij}^d + \sum_{\{i,j\} \in A^d} y_{ij}^d = \sum_{k \in K^d(i,d)} x_k^d \quad \forall d \in D, \forall i \in N^d
\]

\[
y_{i,j}^d + \sum_{\{i,j\} \in A^d} y_{ij}^d = \sum_{k \in K^d(i')} x_k^d \quad \forall d \in D, \forall (i,j) \in A^d
\]

\[
x_k^d, y_{ij}^d \in \{0,1\} \quad \forall d \in D, \forall k \in K^d, \forall (i,j) \in A^d
\]

D – set of all depots
N – set of all tasks
\( N^d \) – set of all tasks of depot \( d \)

\( A^{sd} \) – set of all short edges of depot \( d \)
\( A^d \) – set of all long edges of depot \( d \)

\( y_{ij} \) – edge connecting task \( i \) and \( j \)

Huisman et al. 2005
MDVCSP: Time-Space Network Formulation

\[ \min \sum_{d \in D} \sum_{(i, j) \in A^d} c_{ij}^d y_{ij}^d + \sum_{d \in D} \sum_{k \in K^d} f_k^d x_k^d \]

s. t.:

\[ \sum_{\{i,i,j\} \in A^d} y_{ij}^d = \sum_{\{i,i,j\} \in A^d} y_{ji}^d \quad \forall d \in D, \forall j \in V^d \]

\[ \sum_{d \in D} \sum_{(i, j) \in A^d (t)} y_{ij}^d = 1 \quad \forall t \in T \]

\[ \sum_{k \in K^d (i,j)} x_k^d = y_{ij}^d \quad \forall d \in D, \forall (i, j) \in A^d \]

\[ x_k^d \in \{0,1\} \]

vehicle costs of arc \((i,j)\) in depot \(d\)

flow on arc \((i,j)\) in depot \(d\)

costs of duty \(k\) in depot \(d\)

equals 1 if duty \(k\) in depot \(d\) is selected

Vehicle scheduling

Crew scheduling

+ Linking

\[ 0 \leq y_{ij}^d \leq u_{ij}^d \quad \forall d \in D, \forall (i, j) \in A^d \]

\( y_{ij}^d \) integer \( \forall d \in D, \forall (i, j) \in A^d \)

\( D \) – set of all depots

\( A^d \) – set of productive arcs depot \(d\)

\( y_{ij}^d \) – edge connecting task \(i\) and \(j\)

\( t \) – trip

\( V^d \) – set of nodes

Suhl, Steinzen et al. 2010
Comparison of TSN with Connection-based Formulation

• TSN: More compact formulation; smaller network
  MIP is smaller and easier to solve

<table>
<thead>
<tr>
<th>#arcs/#trips</th>
<th>100</th>
<th>200</th>
<th>400</th>
<th>800</th>
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<td>Connection-based</td>
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<td></td>
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<tr>
<td>network</td>
<td>17800</td>
<td>69500</td>
<td>27300</td>
<td>107500</td>
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<tr>
<td>Time-space</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>network</td>
<td>3000</td>
<td>6500</td>
<td>13800</td>
<td>27900</td>
</tr>
<tr>
<td>% of conn-based</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16,9</td>
<td>9,3</td>
<td>5,1</td>
<td>2,6</td>
</tr>
</tbody>
</table>

Avg. results for Huisman 2005 test set, 10 instances per group
Solution using the TSN formulation

Column generation in combination with Lagrangean relaxation

- Solve MDVSP and CSP sequentially

- Compute dual multipliers by solving Lagrangean dual problem with current set of columns

- Add duties to MDVCS

- Find integer solution

**Approach:**
- Standard MIP-Solver
- Network optimizer
- Heuristics
- Duty generation alg.
Modeling the Column Generation Pricing Problem

• In the column generation phase, we need to generate duties with negative reduced costs
  – a very complex problem with huge degree of freedom

• Usually formulated as a resource constrained shortest path problem (RCSP)

• Define network G(N,A)
  – nodes N: relief points, source, sink
  – arcs A: tasks, task connections (e.g. breaks, deadheads, sign-on/off)

• Duty constraints and piece of work related constraints have to be considered
Network Models for a Decomposed Pricing Problem

Piece generation network

- pieces of work

connection-based duty
generation network
(Freling et al. 1997, 2003)

network size: $O(#\text{tasks}^4)$

aggregated time-space
duty generation network
(Steinzen/Suhl 2011)

network size: $O(#\text{tasks}^2)$
### Computational Results

#### Duty Types with two pieces of work, four depots

<table>
<thead>
<tr>
<th>trips</th>
<th>80</th>
<th>100</th>
<th>160</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>#col.gen. iterations</td>
<td>15.3</td>
<td>19.2</td>
<td>23.5</td>
<td>24.9</td>
</tr>
<tr>
<td>cpu total (hh:min)</td>
<td>00:06</td>
<td>00:13</td>
<td>00:27</td>
<td>01:15</td>
</tr>
<tr>
<td>#blocks</td>
<td>9.2</td>
<td>11.0</td>
<td>14.8</td>
<td>18.4</td>
</tr>
<tr>
<td>#duties</td>
<td>19.7</td>
<td>23.1</td>
<td>32.6</td>
<td>39.3</td>
</tr>
<tr>
<td><strong>Time-Space Network</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Integrated approach total</strong></td>
<td>28.9</td>
<td>34.1</td>
<td>47.2</td>
<td>57.7</td>
</tr>
<tr>
<td><strong>Conn.-based integrated total</strong></td>
<td>29.6</td>
<td>36.2</td>
<td>49.5</td>
<td>60.4</td>
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<tr>
<td><strong>Sequential approach total</strong></td>
<td>35.0</td>
<td>40.9</td>
<td>53.6</td>
<td>65.5</td>
</tr>
</tbody>
</table>

5 duty types with ≤2 pieces of work, 4 depots (Huisman)
Regularity in Vehicle Schedules

• In a timetable, regular trips are offered every day
• Further individual trips occur irregularly
  – Interest groups, events, school classes etc.
• Many public transit providers prefer as regular vehicle (crew) schedules as possible
• Research question:
• How to achieve/measure regularity in vehicle schedules?
**Generation of regular schedules**

**Basic concepts**

**Daily Regularity (Reference)**
- **Input:**
  - Regular trips
  - Irregular trips on one day
  - Reference schedule
- **Goal:** Find a schedule that is similar to the reference schedule

**Regularity over Several Days (Pattern)**
- **Input:**
  - Regular trips
  - Irregular trips of all days
- **Goal:** Find patterns that can be used on several days

 [+ ] less complex problem
[ − ] Similarity depends on the reference plan

[- ] Higher problem complexity
[ + ] Similarity is not limited by reference schedule
Planning Process in Public Transit Networks

Crew rostering problem
- Assign all possible activities to crews, including crew duties, planned reserves, days-off etc. for a given planning period
- Complex work regulations should be held
- Fairness among all drivers
- Preferences of drivers
- Fixed activities (fixed in previous planning period, leaves)

Crew rostering steps:
- Days-off
- Shifts
- Duties
The Crew Rostering Problem in Public Transit
Cyclic and non-cyclic crew rostering

- Cyclic crew rostering problem (CCR)
  - considers days of the week
  - A roster is generated for a group of drivers
  - Preferences are considered for a day of the week
  - Popular and unpopular duties as well as the days-off and weekends-off are evenly distributed
  - Shortcomings:
    - not flexible enough to respond to changes in traffic (special events)

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<td>MS</td>
<td>MS</td>
<td>F</td>
<td>F</td>
<td>ES</td>
<td>ES</td>
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<tr>
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<td>ES</td>
<td>F</td>
<td>F</td>
<td>LS</td>
<td>LS</td>
<td>LS</td>
</tr>
</tbody>
</table>

- Shortcomings:
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<tbody>
<tr>
<td>MS</td>
<td>MS</td>
<td>F</td>
<td>F</td>
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<td>ES</td>
<td>ES</td>
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<tr>
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<td>ES</td>
<td>F</td>
<td>F</td>
<td>LS</td>
<td>LS</td>
<td>LS</td>
<td>LS</td>
</tr>
</tbody>
</table>

ES: early shift
MS: midday shift
LS: late shift
F: day off
The Crew Rostering Problem in Public Transit
Cyclic and non-cyclic crew rostering

- Non-cyclic crew rostering problem (NCCR)
  - considers calendar dates
  - A roster is generated for each driver
  - Preferences can be specifically defined for a calendar date
  - Real traffic schedule every calendar date is considered

<table>
<thead>
<tr>
<th></th>
<th>26.06</th>
<th>27.06</th>
<th>28.06</th>
<th>29.06</th>
<th>30.06</th>
<th>01.07</th>
<th>02.07</th>
<th>03.07</th>
<th>04.07</th>
<th>05.07</th>
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<td>F</td>
<td>F</td>
<td>ES</td>
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<td>d2</td>
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<td>MS</td>
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<td>MS</td>
<td>MS</td>
<td>MS</td>
<td>ES</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

ES: early shift  
MS: midday shift  
LS: late shift  
F: day off

Solution:
Exact solver  
Column generation  
Simulated annealing
Multiobj. metaheur.

Optimization model
### Cyclic and non-cyclic crew rostering

#### Computational results (sequential vs. Integrated)

<table>
<thead>
<tr>
<th>Instance</th>
<th>Unassigned duties (%)</th>
<th>Unassigned days (%)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Sequential approach</td>
<td>Integrated approach</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sequential approach</td>
</tr>
<tr>
<td>48-75-6</td>
<td>1.4</td>
<td>0.3</td>
</tr>
<tr>
<td>52-73-6</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>52-75-6</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>9-238-11 (CCR)</td>
<td>6.3</td>
<td>1.5</td>
</tr>
<tr>
<td>393-45-37</td>
<td>8.6</td>
<td>4.4</td>
</tr>
<tr>
<td>392-45-37</td>
<td>16.9</td>
<td>11.1</td>
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<tr>
<td>397-40-37</td>
<td>9</td>
<td>3.8</td>
</tr>
<tr>
<td>96-70-8</td>
<td>11.7</td>
<td>6.3</td>
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<td>0</td>
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<td>607-70-26</td>
<td>6.3</td>
<td>0.29</td>
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</tbody>
</table>
Decision Support for Crew Rostering
Rota scheduling: computational results with multi-objective metaheuristics
Some References


Conclusion

• Requirements from enterprises often imply challenging research problems for which no solutions exist yet.

• In the optimization area, resulting new models and methods improve the state-of-the-art and can be published in scientific research journals.

• Simultaneously the results have significant practical influence:
  – New models and methods make high cost savings possible.

• Working with practical problems and data often takes lot of time.

• Such time aspects should be appreciated in universities:
  – Not just counting publications, but also impact in practice.
Thank you very much for your attention