Krylov Subspace Methods for Solving Large Linear Systems and Eigenvalue Problems

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Introduction

Very large linear algebra problems arise in the modeling and analysis of many scientific problems, such as stability of mechanical structures, plasma physics, resonance problems in acoustics, quantum chromodynamics, chemical reaction studies, etc. In many cases, standard direct methods can not be used to solve these problems. The systems of equations are sparse, and this structure has to be exploited to get a manageable computational problem. In such cases, Krylov subspace methods can be applied. In this talk we consider techniques for solving large, sparse linear systems and eigenvalue problems.

Iterative methods for linear systems

Smaller systems of equations can be solved in $O(n^3)$ complexity using standard direct methods exploiting LU (or similar) factorizations. Some large, sparse systems can also be successfully solved using sparse direct solvers (implemented in library routines). However for very large, sparse systems an iterative scheme must normally be used.

A short overview of the class of Krylov subspace iterative methods is given, and a few schemes are pointed out to be suitable for different problem types. Also, a few "black-box" preconditioners and more problem-specific preconditioners are considered, including multigrid schemes.

Iterative methods for eigenproblems

Smaller eigenvalue and eigenvector computations, i.e. for matrices of order at most a few thousands can be done with the standard QR-iteration. This requires $O(n^3)$ arithmetic operations which puts a limit on n, apart from storage considerations.

For very large (sparse) problems one needs iterative methods to compute typically only a small fraction of eigenvalues located in a prescribed region of the complex plane. Possibly also the respective eigenvectors need to be computed. To this end one uses the methods of Lanczos, Arnoldi, Davidson and Jacobi-Davidson. Short overview of these methods is given.

Possible topics for informal sessions

- PDE-based preconditioning
- Multigrid methods and multigrid preconditioning
- Preconditioning the eigenvalue problem

References

[1] R. Barrett, M. Berry, T. F. Chan, J. Demmel, J. Donato, J. Dongarra, V. Eijkhout, R. Pozo, C. Romine, and H. Van der Vorst, Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods, 2nd Edition, SIAM, 1994 http://netlib2.cs.utk.edu/linalg/html_templates/Templates.html

[2] Z. Bai, J. Demmel, J. Dongarra, A. Ruhe, and H. van der Vorst, Templates for the Solution of Algebraic Eigenvalue Problems: A Practical Guide. SIAM, 2000 http://www.cs.ucdavis.edu/~bai/ET/contents.html