Challenges in electrohydrodynamic stability analysis of two-phase flows in confining microsystems

Henrik Bruus
MIC - Department of Micro and Nanotechnology, DTU bldg. 345 Ø
Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark

In modern lab-on-a-chip devices electric fields are often employed to control the motion of either the liquids or dissolved particles inside microchannels. Interface instabilities of two-phase flows have a big practical impact in many microfluidic applications. While electrokinetic pumps and devices for liquid-liquid extraction require operational stability, devices for mixing and dispensing would benefit from flow instabilities, particularly because of the low-Reynolds-number regime where one does not expect any turbulence. The study of electrohydrodynamics in confined geometries has therefore become increasingly important the past few years.

Physical model

Electrohydrodynamic stability analysis of two-phase flow in confining microsystems poses some interesting mathematical challenges. The conventional stability analysis for unbounded, viscous two-phase flow [1] must be modified as the system is confined geometrically in microsystems [2,3].

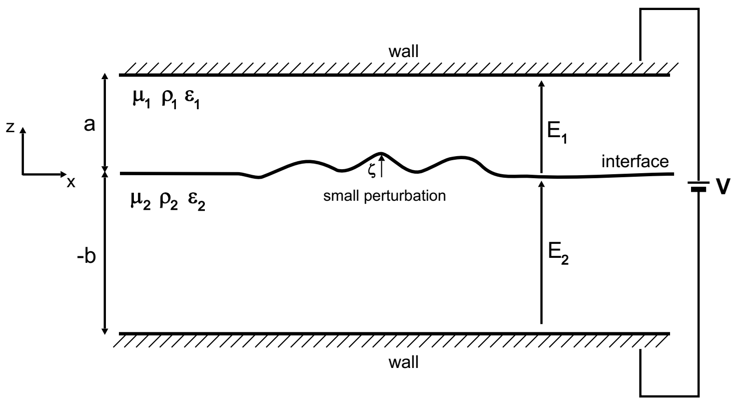

Figure 1: Schematic view of the interface between two liquid dielectrics perturbed in a normal E-field inside a microchannel.

A generic setup is shown in Fig. 1 reprinted from Ref. [3]. Two dielectric liquids with different viscosities $\mu_{1,2}$, densities $\rho_{1,2}$ and dielectric constants $\epsilon_{1,2}$, form an interface described by $z = \zeta(x, y)$ inside a microchannel. The thickness of layer 1 and 2 is a and b, respectively. The interface is influenced by the presence of the external voltage drop V applied across the microchannel.

Our physical model includes the Navier-Stokes and continuity equations, and the

Maxwell equations of electrostatics. In each of two fluids, $i = 1, 2$, we have

$$\rho^{(i)} \frac{D\mathbf{u}^{(i)}}{Dt} = -\boldsymbol{\nabla} p^{(i)} + \mu^{(i)} \nabla^2 \mathbf{u}^{(i)} + \rho^{(i)} + \boldsymbol{\nabla} \cdot T^M, \tag{1}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u}^{(i)} = 0, \tag{2}$$

$$\boldsymbol{\nabla} \cdot (\epsilon^{(i)} \mathbf{E}^{(i)}) = 0, \tag{3}$$

$$\boldsymbol{\nabla} \times \mathbf{E}^{(i)} = 0, \tag{4}$$

where ρ is density, p pressure, μ viscosity, $\epsilon = \epsilon_0 \epsilon_r$ permitivity, $\mathbf{u}(x, y, z) = (u, v, w)$ velocity field, $\mathbf{E}$ electric field, and T^M the Maxwell stress tensor

$$T^M_{ik} = -\frac{\epsilon}{2} E^2 n_i + E_i D_k n_k. \tag{5}$$

The Maxwell stress tensor conveniently describes coupling between electric fields and (fluid) dielectrics. In Eq. (5), $D = \epsilon E$ is the electric displacement vector and $\mathbf{n}$ is a vector normal to the interface. In our analysis we focus on homogeneous perfect dielectrics where the coupling with E fields happens at the interface, directed normally. Thus, the effect of E fields enter only through the normal-stress boundary condition. The boundary conditions are summarized in Ref. [2].

Linear perturbation and stability analysis

In perturbation analysis we expand a total field f (velocity $\mathbf{u}$, pressure p, electric potential ϕ or normal vector $\mathbf{n}$) up to the linear terms by adding a small perturbation f' to the stationary flow solution f_0. When variables $f = f_0 + f'$ are put into the governing equations and boundary conditions, the solution f_0 is subtracted and the equations for f' obtained.

The perturbations are further expanded into normal modes of wave number k and characteristic frequency ω_k: $f' = \hat{f}(z) \exp[i(kx - \omega_k t)]$. If a perturbation frequency $\omega_k = \mathrm{Re}\,\omega_k + i\mathrm{Im}\,\omega_k$ has a positive imaginary part, the disturbance will grow in time as $\exp[\mathrm{Im}\,\omega_k]$, and the system will be unstable.

In case when two streaming liquids are perturbed the convective term $\mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{u}$ will also give a first-order contribution. Furthermore, we restrict our consideration to two-dimensional disturbances, a valid assumption when the primary flow is parallel i.e. when $\mathbf{u}_0(x, y, z) = (u_0(z), 0, 0)$, [2]. The perturbations can then be expressed using the stream function $\psi(x, z)$, and the equations that govern perturbations reduce to single ordinary differential (Orr-Sommerfeld) equation for each liquid

$$\frac{d^4\psi_1}{dz^4} - 2k^2 \frac{d^2\psi_1}{dz^2} + k^4 \psi_1 = ikRe \left[(U_1 - \frac{\omega}{k})(\frac{d^2\psi_1}{dz^2} - k^2\psi_1) - \frac{d^2 U_1}{dz^2} \psi_1 \right], \tag{6}$$

$$\frac{d^4\psi_2}{dz^4} - 2k^2 \frac{d^2\psi_2}{dz^2} + k^4 \psi_2 = i\frac{r}{m} kRe \left[(U_2 - \frac{\omega}{k})(\frac{d^2\psi_2}{dz^2} - k^2\psi_2) - \frac{d^2 U_2}{dz^2} \psi_2 \right]. \tag{7}$$

In Eqs. 6 and 7, $Re = \rho_1 U_0 a / \mu_1$ is Reynolds number (see Fig. 1), U_1 and U_2 are primary flow velocities in fluids 1 and 2 respectively, $m = \mu_2/\mu_1$ is the viscosity ratio and $r = \rho_2/\rho_1$ the density ratio of the two fluids. As mentioned above, electric effects appear solely in the condition for balance of normal stresses.

The challenge

The challenge is to study how, within linear stability analysis, the confinement given by the liquid layer thicknesses a and b influence the stability of the interface as compared to the unbounded system, where a and b tend to infinity.

References

[1] C.-S. Yih, J. Fluid. Mech, **27**, p. 337-352, (1967)
[2] G. Goranović, PhD thesis, Technical University of Denmark, 2003
http://www.mic.dtu.dk/research/MIFTS
[3] G. Goranovic, M.P. Sørensen, M. Brøns, and H. Bruus muTAS-2004, Malmö, Sweden, September 2004, proc. vol. **1**, p. 617-619.