LIST OF SUBMISSIONS FOR AGENT FORUM 2021

Wednesday, October 27th

11:00 Jonas Tölle (HY): *Variability of functions, compositions and differential equations with BV-coefficients*:

Abstract: In stochastic analysis, it is well-established to interpret a differential system in integrated form, a viewpoint conceptually strongly related to the distributional formulation of partial differential equations. However, there are many situations, where even the concept of the integral is subtle. Several powerful theories have emerged to treat these situations, such as rough path theory or the theory of regularity structures. On the other hand, these methods are usually applied to situations where the coefficient maps are smooth, and most of the existing methods break down completely if one allows discontinuities (provided that the forcing term is not too regular). In particular, this is the case if we admit general functions of bounded variation (BV-functions) as a possible choice of our nonlinear coefficients.

In this talk, we combine tools from fractional calculus and harmonic analysis, together with certain fine properties of BV-functions, allowing us to give a meaningful definition for (multidimensional) generalized Lebesgue-Stieltjes integrals for sufficiently regular Hölder functions. The key idea is that the unknown function should not spend too much time on the ""bad"" regions of the BV-coefficient maps. Our novel multiplicative composition estimate leads to a systematic way to quantify this in terms of potential theory of Riesz energies and the occupation measure of the unknown function. We discuss several consequences, and provide existence and uniqueness results for certain differential systems involving BV-coefficients.

While all the results are given in a purely analytic fashion, we give several examples related to stochastic differential equations, with focus on the fractional Brownian motion.

Finally, extensions to functions from fractional Sobolev classes, the application of relaxation methods from variational analysis and first steps toward solving nonlocal and nonlinear PDEs on domains are discussed.

The talk is based on joint works together with: Michael Hinz (Bielefeld University) & Lauri Viitasaari (Uppsala University).

11:20 Cintia Pacchiano (Aalto): *Existence of parabolic minimizers to the total variation flow on metric measure spaces*:

Abstract: In this talk we discuss some fine properties and existence of the variational solutions to the Total Variation Flow. Instead of the classical Euclidean setting, we intend to work in the general setting of metric measure spaces. During the past two decades, a theory of Sobolev functions and BV functions has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in

various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc. The total variation flow can be understood as a process diminishing the total variation using the gradient descent method. This idea can be reformulated using parabolic minimizers, and it also gives rise to a definition of variational solutions. The advantages of the approach using a minimization formulation include much better convergence and stability properties. This is a very essential advantage as the solutions naturally lie only in the space of BV functions. We give an existence proof for variational solutions u associated to the total variation flow. Here, the functions being considered are defined on a metric measure space. For such parabolic minimizers that coincide with a time-independent Cauchy-Dirichlet datum on the parabolic boundary of a space-time-cylinder, we prove existence in a weak parabolic function space. In this paper, we generalize results from a previous work by Bögelein, Duzaar and Marcellini and argue completely on a variational level. This is a join project with Vito Buffa and Michael Collins, from Friedrich-Alexander-Universität Erlangen-Nürnberg.

13:15 Ekaterina Mukoseeva (HY): *Second variation techniques for stability in geometric inequalities*:

Abstract: We study stability of minimizers for several geometric problems. Applying second variation techniques and some free boundary regularity results we are able to prove sharp quantitative isocapacitary inequality, both in the case of standard capacity and that of p-capacity. With the same approach we deduce that charged liquid droplets minimizing Debye-Huckel-type free energy are spherical in the small charge regime.

13:35 Carlos Mudarra (JYU): *Differentiable extensions with uniformly continuous derivatives*:

Abstract: Let w be a modulus of continuity. We characterize the 1-jets (f, G) defined on subsets of a Hilbert space X admitting an extension (F, DF) for some F of class $C^{1,w}(X)$. The function F is given by an explicit formula, and the norm of the extension operator has almost sharp bounds, which are dimension-free in the particular case $X = \mathbb{R}^n$. In addition, if f is bounded (resp. G is bounded), then so is F (resp. F is Lipschitz). Also, (F, DF) depends continuously on the given data (f, G). Similar results are true on superreflexive Banach spaces. This is joint work with Daniel Azagra.

13:55 Gohar Aleksanyan (HY): *Quantitative homogenization of the obstacle problem*:

Abstract: The obstacle problem in a divergent form with sufficiently regular coefficients has been well studied. In the upcoming manuscript, jointly with Tuomo Kuusi, we investigate the obstacle problem with merely bounded measurable coefficients via homogenization. In this talk I shall discuss the stochastic homogenization of the obstacle problem, and the large-scale regularity both for the solution and

for the free boundary.

14:15 Saara Sarsa (HY): *Second order Sobolev-regularity of p-harmonic functions*:

Abstract: Solutions to the homogeneous p-Laplacian equation are called p-harmonic functions. It is well known - since the end of 80's - that certain nonlinear transformations of the gradient of a p-harmonic function have square integrable derivative. In this talk I will discuss on some recent extensions of this result.

15:15 Emma-Karoliina Kurki (Aalto): *Characterizations of weak reverse Hölder inequalities on metric spaces*:

Abstract: We look into different ways to characterize functions that satisfy a weak reverse Hölder inequality on an open subset of a metric measure space with a doubling measure. Among others, they can be described as weak A-infinity weights, a generalization of Muckenhoupt weights that allows for nondoubling weights. While several of our results parallel the theory of Muckenhoupt weights, there are exceptions as well. The talk is based on a joint work with Juha Kinnunen and Carlos Mudarra.

15:35 Maryam Samavaki (UEF): *Navier-Stokes equations on Riemannian manifolds*:

Abstract: The Navier-Stokes equations are dominant on plenty of natural hydrodynamic phenomena and by solving this system we can obtain parameters such as pressure, velocity, and water level in the desired phenomenon. I studied the Navier-Stokes equations on compact Riemannian manifolds and came up with a set of conclusions that link the bahavior of solutions to the geometry of the manifolds. The motivation for such a formulation comes from atmospheric models as well as some thin film flows on curved surfaces. More precisely, I studied the properties of solutions in the presence of Killing vector fields. My main results concerned the decomposition of the flows to the Killing component and its orthogonal complement. Moreover, it turns out that one can produce new solutions with a Lie bracket. Given a solution to a linearized system and a Killing field, their bracket is also a solution to the linearized system. This is rather a technical result where I show that various differential operators behave well with respect to the bracket operation when one of the fields is a Killing field. Finally, I concentrate at the two-dimensional case, which has its own characteristics, as well as the Coriolis effect, which is important in atmospheric flows. I have got the Young Research Award UEF 2021 on my research work.

15:55 Joona Oikarinen (HY): *(Liouville) Conformal Field Theory*:

Abstract: I will explain how to define a conformal field theory in the path integral approach to quantum field theory. Time permitting, I will review some recent results achieved in the path integral approach to Liouville conformal field theory.

16:15 Nikolay Barashkov (HY): *Stochastic control in Constructive Field Theory*:

Abstract: I will describe a stochastic control interpretation of the partition function of Euclidean Field theories(EQFT's), which is based on the Boue-Dupuis formula. Using this interpetation it is possible to prove existence of some EQFT's. In some cases it is also possible to study the correlation functions. One can also give an explicit description of the Laplace transform.

Thursday, October 28th

9:30 Tuomas Oikarinen (HY): *Commutator lower bounds*:

Abstract: We discuss the approximate weak factorization argument that allows to bound several oscillatory norms such as bmo from above with commutators of singular integral operators. The main gist of the argument is given with extensions and results in bi-parameter, bilinear and parabolic settings.

9:50 Emiel Lorist (HY): *Singular stochastic integral operators*:

Abstract: Singular integral operators play a prominent role in harmonic analysis. By replacing the integration with respect to some measure by integration with respect to Brownian motion, one obtains a stochastic singular integral operator. In this talk I will introduce these operators and explain Calderon-Zygmund theory developed for these operators by Mark Veraar and myself.

10:10 Damian Dąbrowski (JY): *On measures with L^2 bounded Riesz transform: to AD regularity and beyond!*:

Abstract: The measures which define an L^2 bounded n-dimensional Riesz transform have been intensely studied in the last 50 years. Especially in the case of n-AD-regular measures (that is, measures that are "quantitatively n-dimensional") the situation is very well understood by now. In this talk I will describe how some of the recent advances in the field can be pushed beyond the AD-regular setting. Based on joint work with Xavier Tolsa.

11:00 Emil Airta (HY): *Two-weight commutator upper bounds*:

Abstract: I give an overview of the recent developments in two-weight upper bound estimates of Calderón-Zygmund commutators. Especially, I focus on extensions in the multi-parameter and multilinear frameworks.

11:20 Jaakko Sinko (HY): *A necessary condition for compactness of commutators*:

Abstract: Commutators of singular integrals and pointwise multipliers have many applications in analysis. Therefore it is important to understand their mapping properties, such as boundedness and compactness, on different function spaces. The systematic study of the boundedness of commutators was started by Coifman,

Rochberg and Weiss (1976). Recently, a fairly complete picture of this question has been obtained by Hytönen (2018). The study of compactness of these operators was started by Uchiyama (1978). The aim is to achieve the same level of generality in the characterization of compactness of commutators as in the recent boundedness results. We provide steps towards this goal by presenting a necessary condition for compactness in a setting that tries to capture the different commutators that have been individually studied before.

13:15 Aleksi Pyörälä (OY): *Covering self-similar sets by tubes*:

Abstract: We say that a set on the plane is tube-null if it can be covered by straight strips of arbitrarily small total width. Among tube-null sets are trivially inculded those of finite one-dimensional Hausdorff measure, while a set of positive two-dimensional Hausdorff measure is certainly not tube-null. However, determining whether a given set that does not satisfy either of these two conditions is tube-null turns out to be a rather challenging problem. Only a few examples of non-trivially tube-null sets on the plane are currently known, namely the Von Koch snowflake curve and the Sierpinski carpet, as well as other non-trivial carpet-like sets obtained with a similar N-adic construction. The tube-nullity of the snowflake curve is due to V. Harangi, while the tube-nullity of the Sierpinski carpet is a product of our recent joint work with P. Shmerkin, V. Suomala and M. Wu. My talk is based on this joint work, with emphasis on why self-similar sets in particular should express "nice" behaviour under covering by tubes, and how one can utilize ergodic theory in studying this kind of geometric property.

13:35 Zhi-Yi Wu (OY): *Beurling dimension and singular measures*:

Abstract: I will give a review on Beurling dimension problems in fractal spectral theory.

13:55 Sebastiano Nicolussi-Golo (JY): *Horizontal jet spaces on Carnot groups*:

Abstract: Jet spaces are fiber bunldes endowed with a contact structure. They have been invented to treat high order derivatives on manifolds and to apply Lie and Cartan methods to study PDEs. In addition, jet spaces on $\mathbb{R}^n$ have been shown to have a natural structure of Carnot groups. Starting from a Carnot group and working only with horizontal derivatives, we construct a certain type of jet space which we may call a horizontal jet space. We prove that horizontal jet spaces on abelian Carnot groups are the standard jet spaces, and that horizontal jet spaces are themselves Carnot groups. We also prove a Backlund type theorem regarding prolongation of contact mapping of horizontal jet spaces. Other applications will also be presented.

14:15 Yu-Liang Wu (OY): *Topological entropy of shift spaces on trees*:

Abstract: The talk will focus on the recent development of the topological entropy of shift spaces on trees, which is related to its measure-theoretical counterpart over Markov random fields on trees. These two fields of research are especially intimate when dealing with the traditional one-dimensional shift spaces, whose relation is described quantitatively by the so-called variational principle. In this talk, the emphasis is put on the existence of the limit of the topological entropy. The state-of-the-art results successfully establish the existence of the limit of the measure-theoretical entropy in the latter case, while the study on the limit of the topological entropy over is limited. For this talk, a brief introduction of the shift spaces on trees (including semigroups, free groups, Bethe lattices, and exploding trees) as well as a short discussion of the interplay between deterministic systems (shift spaces) and random systems (Markov random fields) is carried out, and the sufficient conditions and sketches of proofs are provided for the existence of limit in the topological entropy. Also, the question should be posed in the talk whether the discussions above could be generalized to the pressure over shift spaces.

15:15 Miguel García Bravo (JY): *Bi-Lipschitz invariance of $W^{1,1}-$ and BV-extension domains*:

Abstract: We will present here some new results about Sobolev extension domains and BV extension domains. It is an interesting question if the property of being a Sobolev extension domain is preserved under bi-Lipschitz homeomorphisms. For the case p>1 this was solved in the positive by Hajłasz, Koskela and Tuominen in 2008 by using the fact that the Hajłasz Sobolev space and the usual Sobolev space coincide in this case. For the case p=1 the result is known to still hold for bounded simply connected planar domains. In this talk we will show how to slightly generalize this result by proving the bi-Lipschitz invariance of planar bounded $W^{1,1}$-extension domains. We do this by first resolving the question for BV-functions and then by applying a recent characterization of $W^{1,1}$-extension domains among bounded BV-extension domains.

15:35 Ugo Bindini (JY): *Optimal Transport and Density Functional Theory*:

Abstract: I will present the theory of multi-marginal Optimal Transport, and how this can be applied to Density Functional Theory in the study of molecular structures.

Friday, October 29th

9:30 Thavamani Govindaraj (TY): *A Finite Dimensional Controller for a Flexible Satellite*:

Abstract: In this talk, we consider output tracking problem of a flexible satellite which is composed of two symmetrical flexible solar panels and a center rigid body. We discuss exponential stability of the model using linear semigroup theory. We

present a low-gain controller that achieves robust output tracking of given combinations of sinusoidal signals.

9:50 Juhani Nissilä (OY): *Fourier decay of absolutely and Hölder continuous functions with finitely or infinitely many oscillations and error analysis of their numerical Weyl fractional derivatives*:

Abstract: Suppose that a function is absolutely continuous and uniformly Hölder continuous and that its finite difference function does not oscillate infinitely many times on a bounded interval. Then the decay rate of its Fourier coefficients can be estimated exactly. None of the assumptions can be relaxed without weakening the decay for some functions. The uniform Hölder continuity of infinitely oscillating chirps and the decay of their Fourier coefficients are studied. The main result is then connected to the estimation of maximum absolute error of numerical Weyl fractional derivatives calculated using the discrete Fourier transform. This error is bounded by some constant times the sequence length raised to a negative power. The exponent depends on the smoothness of the signal. This contrasts with using difference quotients in numerical differentiation, in which case the error is bounded by a constant times the sequence length raised to a some fixed negative power and the order of the method defines that exponent.

10:10 Henrik Wirzenius (HY): *The quotient Banach algebra of compact-by-approximable operators on Banach spaces*:

Abstract: Let $K(X)$ denote the Banach algebra of compact operators acting on a Banach space X and $A(X)$ the uniform closure of the bounded finite rank operators. In this talk I will describe my PhD research on the quotient algebra $K(X)/A(X)$ of compact-by-approximable operators. This is a radical Banach algebra that is poorly understood, mainly because $K(X)/A(X)$ is non-trivial only within the class of Banach spaces X failing the approximation property.

11:00 Zheng Zhu (JY): *Sobolev functions on cusp domains*:

Abstract: In this talk, I will present results about Sobolev functions on cusp domains. Including Sobolev extension property of outward cusp domains and equivalence of Hajłasz-Sobolev spaces and Sobolev spaces on outward cusp domains.

11:20 Khanh Nguyen (JY): *Limits at infinity of weighted Sobolev functions*:

Abstract: The aim of this talk is to discuss the existence and uniqueness of limits at infinity of weighted Sobolev functions. This is a joint work with Sylvester Eriksson-Bique (Oulu) and Pekka Koskela (Jyväskylä).

13:15 Susanna Heikkilä (HY): *Quasiregular curves of small distortion in product manifolds*:

Abstract: Quasiregular curves extend the notion of quasiregular mappings to the setting, where the range may have larger dimension than the domain. In this talk, I will discuss quasiregular curves from Riemannian n-manifolds into Riemannian product manifolds, where each factor of the product is a Riemannian n-manifold. In particular, I will discuss a recent result stating that, if n is at least three, then such a quasiregular curve of small distortion has a quasiregular coordinate mapping. This is based on joint work with Pekka Pankka (HY) and Eden Prywes (Princeton).

13:35 Toni Ikonen (JY): *Uniformization of metric surfaces*:

Abstract: An aim of the talk is to unpack the following theorem: Given a metric surface Z that can be covered by quasiconformal images of planar domains, there exists a Riemannian surface M and a quasiconformal homeomorphism from M onto Z.

In particular, the statement implies that finding local parametrizations from Riemannian surfaces implies the existence of a global parametrization from such a Riemannian surface (for a suitable notion of a parametrization). We give some examples of spaces for which the result applies and exhibit some limitations of the theorem.

13:55 Arttu Karppinen (UTU): *Sharp growth conditions for maximal function in generalized Orlicz spaces*:

Abstract: This talk combines Hardy-Littlewood maximal function and spaces related to non-standard growth conditions used in study of partial differential equations. It is well known that in Lebesgue spaces the Hardy-Littlewood maximal function is bounded if and only if $p > 1$. This fact has been generalized to Orlicz spaces (generating function has to satisfy ∇_2-condition) and variable exponent spaces (infimum of $p(x)$ must be strictly greater than 1). The purpose of this talk is to investigate this question in generalized Orlicz spaces which contain as special cases previously mentioned Lebesgue, Orlicz and variable exponent spaces among others. Surprisingly, we find that the corresponding assumption regarding the parameter p is not necessary for boundedness of the maximal operator. Examples of various cases are given. This talk is based on joint work with Petteri Harjulehto.

14:15 Timo Takala (Aalto): *The function space JNp*:

Abstract: In this talk I will present the John-Nirenberg space JNp and explain why it is interesting to study. I will go through some of the basic properties of JNp functions. If I have time, I will briefly show my own contributions and some open questions.

14:35 Kim Myyryläinen (Aalto): *Dyadic maximal operator on the dyadic John–Nirenberg space*:

Abstract: It is well-known that the Hardy-Littlewood maximal operator is bounded on the space of functions of bounded mean oscillation (BMO). In this talk, we discuss the boundedness of the dyadic maximal operator on the dyadic John–Nirenberg space which is a generalization of BMO. We provide a method to construct nontrivial functions in the dyadic John–Nirenberg space. Some difficulties in the standard non-dyadic case are also discussed.